NSG Calculation Guidance

This is to be used as a guide for staff, students and parents

Background, Purpose and Aims

Mathematics is a subject in which its learning episodes can be taught in multiple different ways, using multiple different methods; this is the case at many schools. This can cause significant confusion and cognitive overload for some students, especially lower attaining students.

The purpose of this document is to provide mathematics teachers, teachers of other subjects and staff who support students in mathematics lessons with an easy-reference guide to some the methods that could be employed in the teaching of mathematics.

It is also intended to be used as a guide for parents and students to use and apply to different mathematical problems.

In mathematics we aim to ensure thorough understanding of the subject content covered without relying on process memorisation. The methods outlined are one of the ways of accomplishing this.

This document will allow staff to synchronise their practise, to ensure students encounter the same methods throughout their mathematical journey, regardless of their teacher.

The aim is that this will provide consistency for students in the long-term and therefore aid in improving their outcomes.

We acknowledge that there may be other methods of teaching the topics included in this Policy, some of which may work better for other styles of teaching. The methods included are those that we have found beneficial for our students.

The Policy does not include all topics, nor does it include all approaches for all sub-topics. This is the first version of an ongoing project.

Note: This is a working document and aims to take account of all research and best practise. This document does not aim for a 'one size fits all' approach, nor does it intend to stipulate how to teach, it is merely a mechanism to improve the consistency of students' mathematical experience.

Acknowledgements

This document was written and created by Martin Green and Phil Bruce (Co-Creators of Purposeful Maths).
We would also like to acknowledge the input and thoughts of the Maths and Science departments during the implementation of this Policy.

<u>Introduction</u>

Throughout this document each approach is split into four stages: **Physical**, **Pictorial**, **Semi-Abstract** and **Abstract**. The idea is through a systematic approach students will begin, where possible, to explore mathematics by using physical manipulatives so that at the end of the process students should be able to form their own generalisations of mathematical rules.

Here are the four stages, along with explanations of each:

Stage	**Name**	Explanation
1	**Physical**	During the **Physical Stage**, students will have the opportunity to work with manipulatives and other physical objects in order to understand the mathematical concept. There will be times where this is not possible, in these cases students should begin at Pictorial Stage.
2	**Pictorial**	During the **Pictorial Stage**, students should be able to pictorially or diagrammatically represent ideas discovered during the Physical Stage.
3	**Semi-Abstract**	During the **Semi-Abstract Stage**, students should begin to be able to form generalisations and therefore not be fully reliant on a diagram. During this stage, however, a student's understanding should still partly be reliant or aided by a diagram.
4	**Abstract**	During the **Abstract Stage**, students should no longer require a diagram to understand the concept. Students should have formed <u>comprehensive</u> generalisations during which the underlying mathematics is <u>fully</u> <u>understood</u>.

Ideally, by the end of Year 11, all students will fully understand all of the abstract methods taught, along with the underlying mathematics.
Realistically, however, this will not be the case for all students. By the end of Year 11, some students may not grasp the abstract nature of some topics, however this approach allows students to succeed using multiple representation and thus have a greater chance of success.

For example, when expanding single brackets, some lower attaining students may not fully understand why when asked to expand the answer is because the terms inside the brackets need to be multiplied by the term outside. However students may see where this comes from when this is represented using a Semi-Abstract Grid Method.

Language of Mathematics

Students will face similar mathematical problems across the STEM curriculum. The teaching of similar topics across different subjects can pose a problem when different language is used. To minimise the possibility of misconceptions all teachers are committed to modelling the language below to ensure that key content is taught effectively in each curriculum area.

Misconception of Language	Curriculum	Explanation	Shared Language of Mathematics
'Take away / Times'	Maths Science FADT Social Sciences Geography	Across different subjects the words 'take away and times' are used to describe 'subtraction and multiplication'. This can cause RESEARCH	Subtract and Multiply
'Straight Line of Best Fit'	Maths Science FADT Social Sciences Geography	The use of lines of best fit across different subjects poses a problem as in Maths lines are always straight although in Science, Geography and Social Sciences they can appear curved/bent and are still referred to as lines of best fit.	'Lines of best fit can be curved in other Subjects but remain straight in Maths'
'Drawing a Graph'	Maths Science FADT Social Sciences Humanities	In Science, students are expected to draw line graphs and appoint scales based on ranges of data from KS3. In Maths, students are not expected to draw graphs nor appoint scales until KS4.	Teachers must ensure that they model graph drawing effectively and give plenty of practice at KS3.

'Minus Numbers'	Maths Science FADT Social Sciences Humanities	In Maths, students are taught that negative numbers fall below 0 and these can be referred to as minus numbers elsewhere in the curriculum. This poses a misconception when negative numbers are then added or subtracted.	Negative Numbers

Richard Boohan, Richard Needham. (2016). The Language of Mathematics in Science. [Online]. ASE. Last Updated: 2016. Available at: https://www.ase.org.uk/mathsinscience [Accessed 17 September 2024].

Curriculum Area: NUMBER

Addition and Subtraction
Factors
Directed Number
Fractions
Percentages

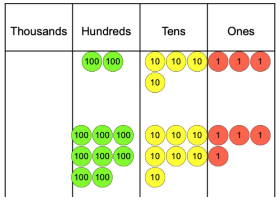
COLUMN ADDITION (INTEGERS AND DECIMALS)

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to use **place value counters** to represent the two numbers and understand the idea of carrying.	Students should draw the manipulatives as a diagram in their books, clearly showing where the values are being carried.	Students should use the ideas in the Physical and Pictorial stages and partition the numbers.	Students should use the traditional column addition method, carrying were appropriate
Example(s)	**Example(s) - These should be seen in books**		

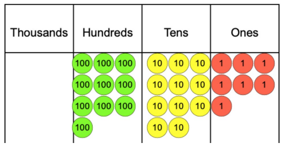
Calculate 243 + 874

Stage 1 Physical

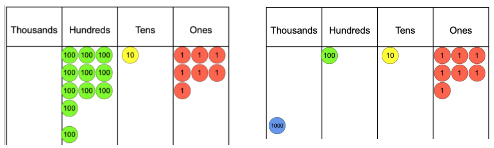
Students should represent the numbers in a place value table using place value counters.



Students should then physically move the counters together starting from the leftmost column.



Students should then understand that 10 1's make 10, 10 10's make 100, etc. and that they can replace these 10 counters with 1 of the next highest counters.



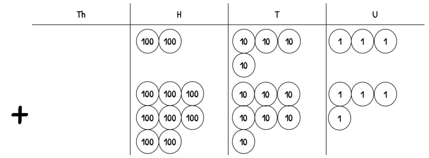
[Notice the added counter is placed lower than the existing ones, this will aid with the idea of carrying later]

Students can then just count the leftover counters.

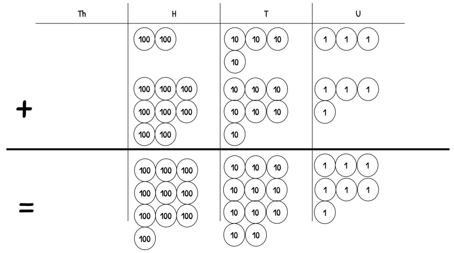
243 + 874 = 1117

Stage 2 Pictorial

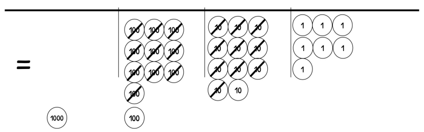
Students should draw the place value table and counters in their books.



Students should then total up each place value column and draw the correct number of each counter.



Where there are more than 10 of a place value counter, students should cross out those which would total the next value up and draw an additional counter in the appropriate column, this should be below those counters that are already there.



Students can then just count the non-crossed out circles.

243 + 874 = 1117

Stage 3 Semi-Abstract

Students should partition the numbers and represent these values in a place value table

	Th	H	T	U
		200	40	3
+				
		800	70	4

Students should the sum each column, carrying where appropriate.

	Th	H	T	U
		200	40	3
+				
		800	70	4
=		1000	110	7
=	1000	100	10	7

Students should then sum the final row.

243 + 874 = 1117

Stage 4 Abstract

Students should be able to use a formal column method of addition

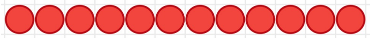
```
   2 4 3
 + 8 7 4
 -------
 1 1 1 7
 1 1
```

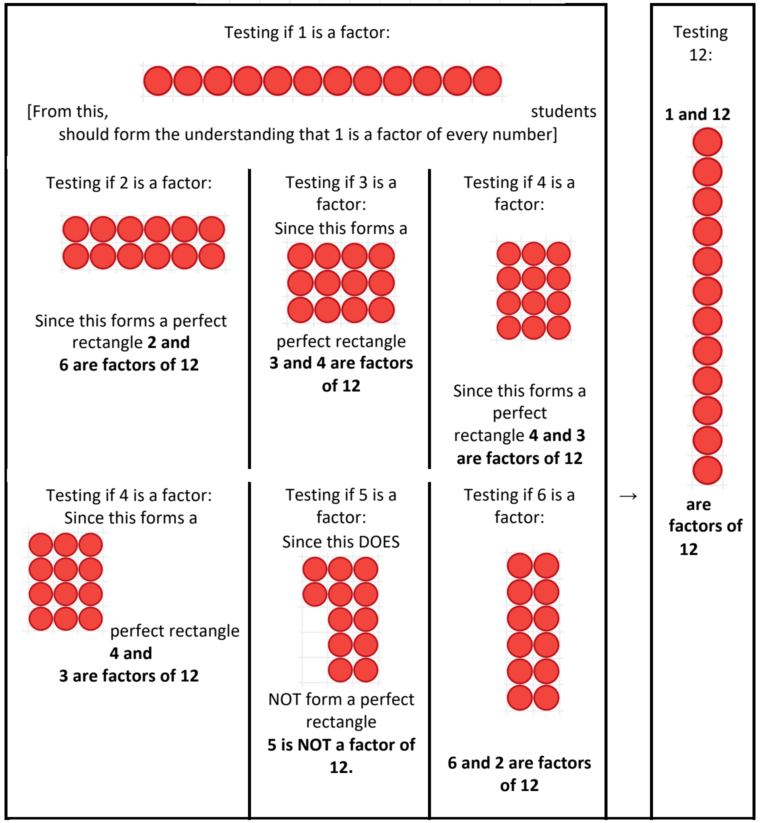
243 + 874 = 1117

FACTORS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should use a manipulative (likely double sided counters or multilink cubes) and maneuver them to understand factors and multiples	Students use the ideas formed in the physical stage to use the squares in exercise books to draw rectangles	Students should use a diagram to find the factors of the required number	Students should be able to list the factors immediately
Example(s)	**Example(s) · These should be seen in books**		
Find all of the factors of 12			

Students should represent the number using the manipulative





Students should identify that the factors come in pairs and that they only
Students should then understand that amount of counters along and the amount of counters up are factors of the value.

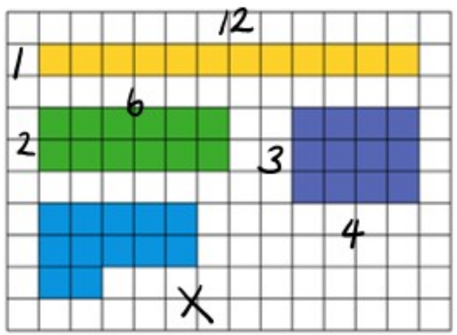
Students should then work **systematically**, adding one to the height and attempting to form a rectangle.

need to be listed once.

Factors of 12 are 1, 2, 3, 4, 6, 12

Students should use the squares in their books to sketch the value in rectangles.

From the Physical stage, students should understand that the factors come in pairs and therefore only need to be drawn once.

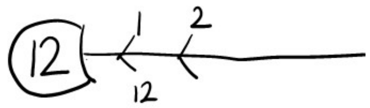


Students should clearly explain why certain values are NOT factors.

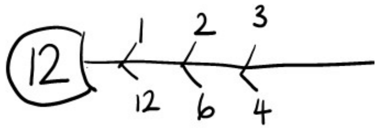
E.g. the light blue diagram above does not form a perfect rectangle, therefore 5 is NOT a factor of 12.

Factors of 12 are 1, 2, 3, 4, 6, 12

Students should work systematically, starting at 1 and increasing by one each time and determining if it is a factor and the corresponding value.



Students should understand that as soon as they reach a number that is already in the diagram they have found all of the factors.



Students MUST then list the factors.

Factors of 12 are 1, 2, 3, 4, 6, 12

Students will not need any form of diagram at this stage.

Students should understand the meaning of factors and will likely be able to find factors in their head.

Factors of 12 are 1, 2, 3, 4, 6, 12

DIRECTED NUMBER

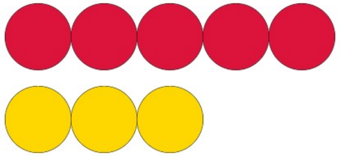
GENERAL TEACHING & LEARNING POINTS	**The following apply to all four operations with negative numbers:** • Teachers should not use the term “minus”, instead use the words “negative” and “subtract” • “Two negatives make a positive” should NEVER be used – this is not mathematically accurate • The idea of zero-pairs should be emphasised throughout as this will also be used in an algebra context • The idea that subtraction is the **additive inverse** should be shown, e.g. subtracting 3 is the same as adding -3 • Students should be confident with using double-sided counters to represent numbers prior to attempting four operations

Addition

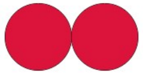
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to use **double-sided counters** to represent the two numbers and the idea of zero pairs.	Students should draw the manipulatives as a diagram in their books, clearly showing the zeropairs.	Students should use the ideas in the Physical and Pictorial stages be able to determine, through using ‘scaled’ double-sided counter diagrams to determine if the answer is positive or negative.	Students should be able to state an answer without the use of a diagram.
Example(s)	**Example(s) - These should be seen in books**		

Calculate -5 + 3
“Negative 5 add 3”

Students should use double-sided counters to represent the -5 using five -1 counters and then physically add three +1 counters.



Students should then identify the zero-pairs in their representation and remove them.



Students should then be able to identify their answer from their remaining counters.

-5 + 3 = -2

Students should represent the calculation as in the Physical Stage, however, for short it may be beneficial for students to use – and + signs instead of drawing full circles.

\- - - - -

\+ + +

Students should then identify the zero-pairs in their representation **and circle them**. They could show they have been removed by crossing through them.

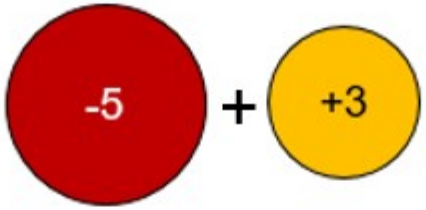
Students should then be able to identify their answer from their remaining symbols.

-5 + 3 = -2

Students should represent this by drawing a larger negative circle plus a smaller positive circle. https://mathsbot.com/manipulatives/directedCounters may be useful for this representation.

For weaker students a number line representation in addition may also be beneficial.

Students should be able to determine from the calculation



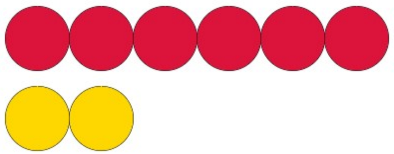
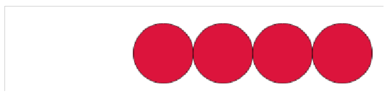
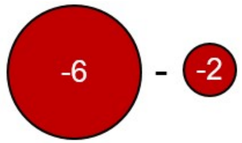
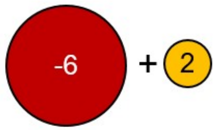
that there are will be more negative counters than positive ones in this calculation, **therefore the answer must be negative.**

-5 + 3 = -2

Students should be able to perform the calculation without any working.

-5 + 3 = -2

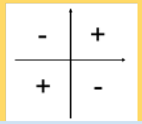
Subtraction

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to use **double-sided counters** to represent the two numbers and the idea of zero pairs.	Students should draw the manipulatives as a diagram in their books, clearly showing the zeropairs.	Students should use the ideas in the Physical and Pictorial stages be able to determine, through using 'scaled' double-sided counter diagrams to determine if the answer is positive or negative.	Students should be able to state an answer without the use of a diagram.
Example(s)	**Example(s) - These should be seen in books**		
Calculate -6 – -2 "Negative 6 subtract -2"			
Students should use double-sided counters to represent the -6. https://mathsbot.com/manipulatives/doubleSidedCounters may be useful for this representation. Students should then understand that subtracting -2 is equivalent to +2. Students should then add these counters to their representation.  Students should then identify the zero-pairs in their representation and remove them.  Students should then be able to identify their answer from their remaining counters. -6 - -2 = -4	Students should represent the calculation as in the Physical Stage, however, for short it may be beneficial for students to use – and + signs instead of drawing full circles. - - - - - - + + Students should then identify the zero-pairs in their representation **and circle them**. They could show they have been removed by crossing through them  Students should then be able to identify their answer from their remaining symbols. -6 - -2 = -4	Students should represent this by drawing a larger negative circle subtract a smaller negative circle. https://mathsbot.com/manipulatives/directedCounters may be useful for this representation. For weaker students a number line representation in addition may also be beneficial.  Students should then apply the additive inverse as they have done in previous stages and re-draw.  Notice the size of the second circle does not change. Students should be able to determine from the calculation that there are will be more negative counters than positive ones in this calculation, **therefore the answer must be negative.** -6 - -2 = -4	Students should be able to perform the calculation without any working. -6 - -2 = -4

Multiplication

GENERAL TEACHING & LEARNING POINTS

- Students should be confident using factors and multiples and the use of a co-ordinate grid.
- Students should understand that the opposite side of the x or y axis involves multiplying by -1 (flipping the counters over)



Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to use **double-sided counters** and a numbered grid to represent the problem as an area model.	Students should draw the manipulatives as a diagram in their books.	Students should use the ideas in the Physical and Pictorial stages be able to determine an answer, through using the area of a rectangle model.	Students should be able to state an answer without the use of a diagram.
Example(s)	**Example(s) - These should be seen in bo ks**		

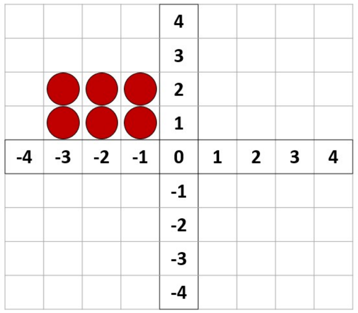
Calculate -3 x 2

"Negative 3 ultiplied by 2"

Stage 1 Physical

For consistency students could represent the first number on the x-axis and the second the y-axis.

Students should use the grid as shown below:



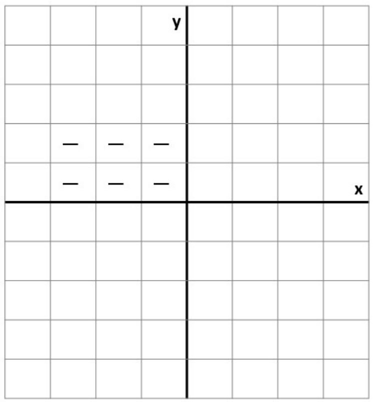
The counters are -3 along the x axis and +2 lots of this (so +2 on the y-axis).

Students should then be able to identify their answer from the counters on their grid

-3 x 2 = -6

Stage 2 Pictorial

Students should draw a small grid in their exercise books and place either + or – symbols in each square to represent the same idea as in the physical stage.

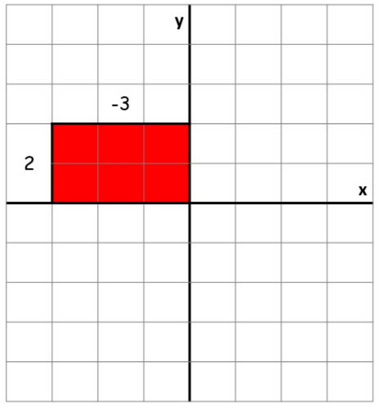


Students should then be able to identify their answer from the signs in their grid

-3 x 2 = -6

Stage 3 Semi-Abstract

Students should draw a similar diagram to those in the Physical and Pictorial stage, however at this stage students should represent the calculation through an area model.



Students should then apply their knowledge of understanding

-3 x 2 = -6

Stage 4 Abstract

Students should be able to perform the calculation without any working.

-3 x 2 = -6

Division

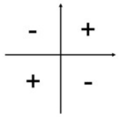
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to use **double-sided counters** and a numbered grid to represent the problem as an area model.	Students should draw the manipulatives as a diagram in their books.	Students should use the ideas in the Physical and Pictorial stages be able to determine an answer, through using the area of a rectangle model.	Students should be able to state an answer without the use of a diagram.
Example(s)	**Example(s) - These should be seen in books**		

Calculate -8 ÷ -2

"Negative 8 divided by negative 2"

Stage 1 Physical

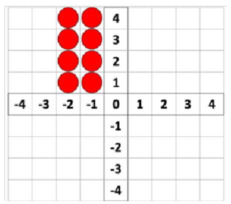
Students will be confident with the + - grid below.

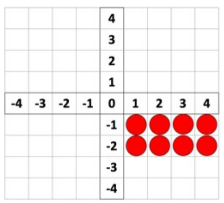


Students should know that for this calculation -8 represents the counters required, and this would be represented in quadrant 2 or 4.

Student should understand that one dimension needs to be -2.

Either representation below could be used.



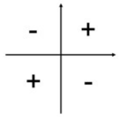


Students should then be able to identify the solution to the problem is the length/width of the other dimension of the diagram.

-8 ÷ -2 = 4

Stage 2 Pictorial

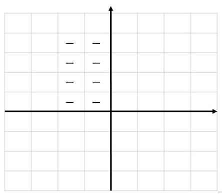
Students will be confident with the + - grid below.

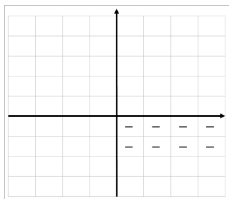


Students should know that for this calculation -8 represents the counters required, and this would be represented in quadrant 2 or 4.

Student should understand that one dimension needs to be -2.

Either representation below could be used.





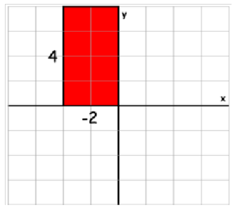
Students should then be able to identify the solution to the problem is the length/width of the other dimension of the diagram.

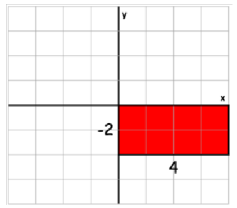
-8 ÷ -2 = 4

Stage 3 Semi-Abstract

Students should represent the calculation as an area as with Dividing Numbers.

Either representation below could be used.





Students should then be able to identify the solution to the problem is the length/width of the other dimension of the diagram.

-8 ÷ -2 = 4

Stage 4 Abstract

At this stage students should be able to perform the calculation without the need for any working.

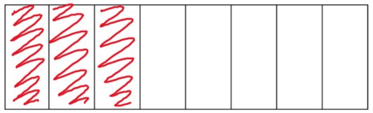
-8 ÷ -2 = 4

EQUIVALENT FRACTIONS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should then be able to represent each fraction as a diagram, one split horizontally and the other vertically	Students should use a scale factor approach to find equivalent fractions.	Students should be able to state equivalent fractions.
Example(s)	**Example(s) - These should be seen in books**		
Write down three fractions that are equivalent to ! "			

This may be added at a later date.

Students should represent the ! fraction using a " diagram, split either horizontally OR vertically.



Students should then understand that as long as this is **equally** split horizontally (or vertically if the original

!
"

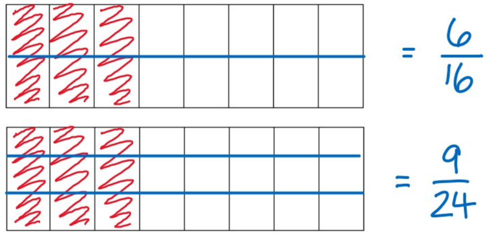
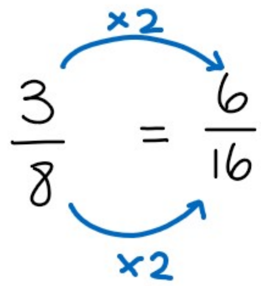


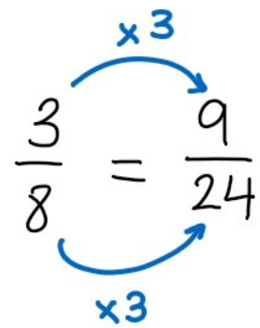
diagram is split horizontally) then the

fractions will be equivalent to

Students should confidently understand that if the numerators and denominators are multiplied or divided by the same factor then the two fractions will be equivalent.

This multiplication should be shown at this stage.





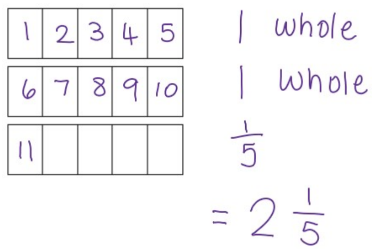
At this stage students should be able to

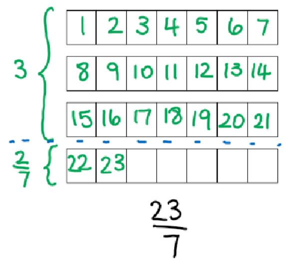
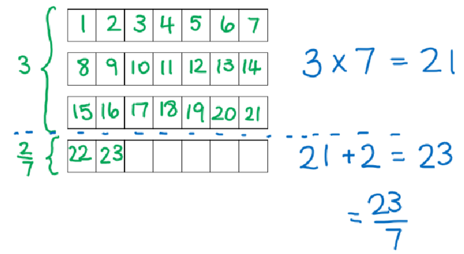
!

immediately state fractions that are equivalent to _

"

CONVERTING BETWEEN MIXED NUMBERS AND IMPROPER FRACTIONS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should represent the initial quantity using a bar diagram.	Students at this stage should be able to determine the number of parts in the wholes plus any extras.	Students should be able to state the equivalent improper fraction/mixed number without any working.
Example(s)	**Example(s) - These should be seen in book**		
Write ##__ as a $ nixed number.			
This may be added at a later date.	Students should understand that, since the denominator is 5, each whole one is split into 5 pieces. Students should draw a diagram to show the 11 parts.  Students should then identify the whole parts and the fraction parts and combine.	Students should be encouraged to think of how many **whole** 5s are in 11, and what the remainder is. $\frac{11}{5} = 2$ remainder 1 $= 2\frac{1}{5}$	At this stage students should be able to immediately state the answer. $\frac{11}{5} = 2\frac{1}{5}$
Write 3%_ as a & nixed number.			

This may be added at a later date.	Students should understand that, since the denominator is 7, each whole one is split into 7 pieces. Students should then count the number of parts. 	Students should **partly** rely on the diagram at this stage. Students should be able to determine the number of sevenths in the three whole ones and add the extras. 	$3\frac{2}{7} = \frac{23}{7}$

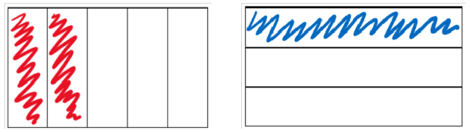
ADDING AND SUBTRACTING FRACTIONS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should then be able to represent each fraction as a diagram, one split horizontally and the other vertically	Students should be able to determine the LCM of the two denominators.	Students should be able to add and subtract fractions without the need for a diagram; understanding the need for the LCM.
Example(s)	**Example(s) - These should be seen in books**		
Calculate %+#– $!			

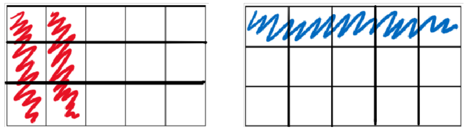
This may be added at a later date.

Students should represent each of the two fractions as separate diagrams.

ONE DIAGRAM SHOULD BE SPLIT VERTICALLY AND THE OTHER HORIZONTALLY.

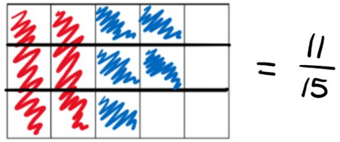


Students should then identify the LCM in order to find a common denominator and split each representation into this number of parts – See Equivalent Fractions.

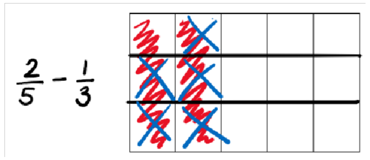


Students should then understand that now all the small parts in the diagram are the same size, these can be combined.

For addition, students should add the parts in both diagrams to a single diagram.

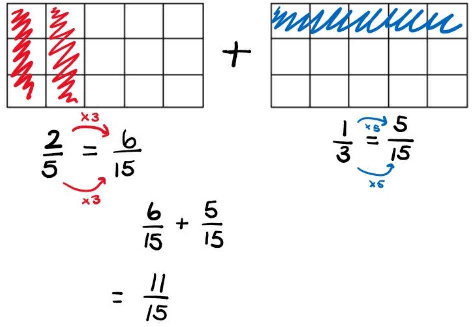


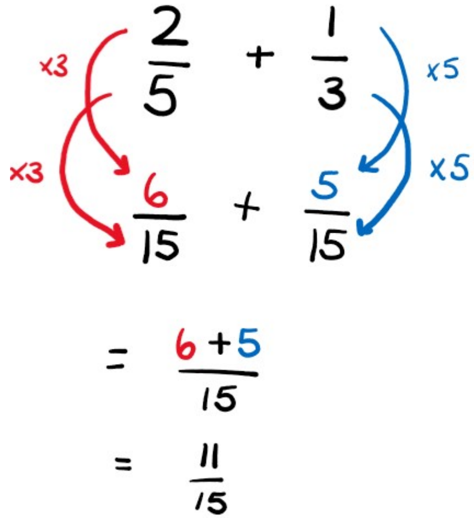
[For subtraction, students should cross through a part in the first diagram for each part in the second diagram]



Students should be able to determine the LCM of the two denominators, 15 in this case.

Students should then draw diagrams to represent these.





FRACTIONS OF AMOUNTS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should be able to represent the problem using a bar model.	Students should use a proportion idea first identifying a unit fraction and then scaling up.	Students should use a simple written method that shows finding a unit fraction and then multiplying.
Example(s)	**Example(s) - These should be seen in books**		

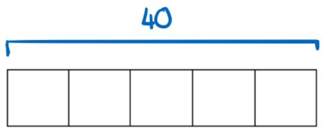
Find ! $ of 40

Stage 1:

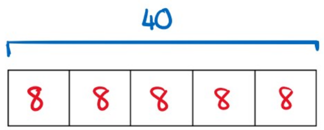
This may be added at a later date.

Stage 2:

Students should represent the problem as a bar model, splitting the whole bar into the number of parts shown in the denominator and clearly showing the total.

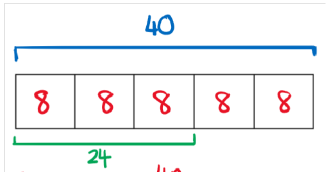


Students should then work out the value of one part, students should then add this to their diagram.



$\frac{1}{5}$ of 40 $= \frac{40}{5} = 8$

Students should then clearly show how they have ! $ obtained the



$\frac{1}{5}$ of 40 $= \frac{40}{5} = 8$

$\frac{3}{5}$ of 40 $= 8 \times 3 = 24$

Stage 3:

Students should find # $ and then multiply this by 3.

$\frac{1}{5}$ of 40 $= \frac{40}{5} = 8$

×3 ↓ ↓ ×3

$\frac{3}{5}$ of 40 $= 24$

Stage 4:

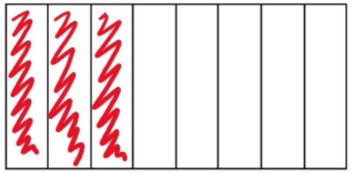
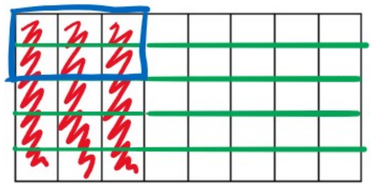
Students should show find # $ and then multiply this by 3 as a single calculation.

$$\frac{40}{5} \times 3 = 24$$

MULTIPLYING FRACTIONS

GENERAL TEACHING & LEARNING POINTS	• To be consistent with diagrams, students should understand that multiplying fractions means the second a fraction OF the first, e.g. # × _# means # of _# $ % % $		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should be able to represent the problem using a bar model.	Students should clearly show their written method, simplifying their answer.	Students at this stage should look for common factors to cancel to avoid larger numerators and denominators.
Example(s)	**Example(s) - These should be seen in books**		

Calcul ıte !×% – " $

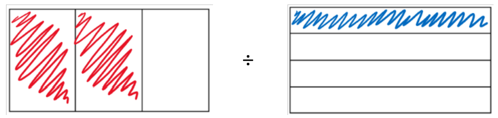
This may be added at a later date.	Students should draw a diagram to represent the ! "  Students should then find % & of the shaded part. Students should divide this into 5 part.  Students should circle the % & $= \frac{6}{40}$ $= \frac{3}{20}$	Students should complete the calculation without the need for a diagram. It is important that students include the !×% "×& step to clearly show their understanding. $\frac{3}{8} \times \frac{2}{5}$ $= \frac{3 \times 2}{8 \times 5}$ $= \frac{6}{40}$ $= \frac{3}{20}$	Students should be encouraged to spot possible calculations after the first step, as shown below. $\frac{3}{8} \times \frac{2}{5}$ $= \frac{3 \times \cancel{2}^{1}}{_{4}\cancel{8} \times 5}$ $= \frac{3 \times 1}{4 \times 5}$ $= \frac{3}{20}$

DIVIDING FRACTIONS

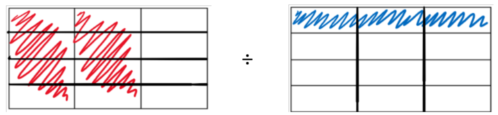
GENERAL TEACHING & LEARNING POINTS	• “Keep Flip Change” should never be used • Students should be encouraged to “multiply by the reciprocal” • Students should be encouraged to think of ÷# as “How many #_ are there in _%” % !’ ’ !		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should be able to represent the problem using a bar model.	Students should clearly show their written method, simplifying their answer.	Students at this stage should clearly show division by a fraction is equivalent to multiplying by its reciprocal.
Example(s)	**Example(s) - These should be seen in books**		
Calculate %÷#_ !’			

This may be added at a later date.

Students should represent each fraction in the calculation as separate diagrams, splitting one vertically and the other horizontally.

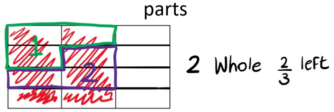


Students should use their knowledge of Equivalent Fractions to split their diagrams so that the parts in each diagram are equal sizes.



Students should understand that students need to determine

Students should then circle the groups of three parts



$= 2\frac{2}{3}$

Students should understand that if they multiply both fractions by the reciprocal of the second, this reduces the calculation to a multiplication, and division by 1.

$$\frac{2}{3} \div \frac{1}{4}$$

$\times\frac{4}{1}$ $\times\frac{4}{1}$

$$\left(\frac{2}{3}\times\frac{4}{1}\right) \div \left(\frac{1}{4}\times\frac{4}{1}\right)$$

$$= \frac{8}{3} \div 1$$

$$= \frac{8}{3}$$

$$= 2\frac{2}{3}$$

Students should represent this multiplication by the reciprocal as a single calculation.

$$\frac{2}{3} \div \frac{1}{4}$$

$$= \frac{2}{3} \times \frac{4}{1}$$

$$= \frac{8}{3}$$

$$= 2\frac{2}{3}$$

PERCENTAGE OF AMOUNTS

GENERAL TEACHING & LEARNING POINTS	•		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should be able to represent the problem using a bar model.	Students should use a written method, **set out using a proportion table**.	Students should use a written, **multiplier**, method in order to calculate the required percentage.
Example(s)	**Example(s) - These should be seen in books**		

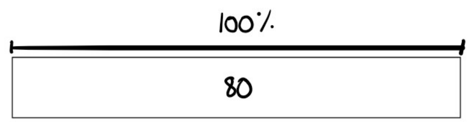
Calculate 35% of 80

Stage 1 Physical

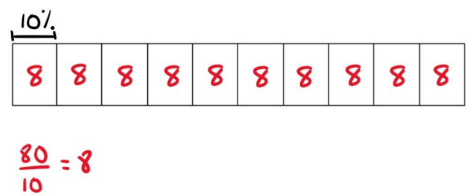
This may be added at a later date.

Stage 2 Pictorial

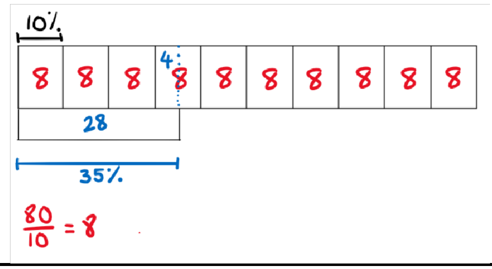
Students should use a bar model to represent the initial 100%.



Students should then split their bar model into a suitable number of parts.

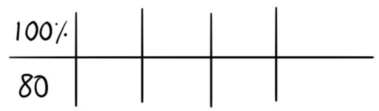


Students should then determine the correct percentage by using a second bar underneath

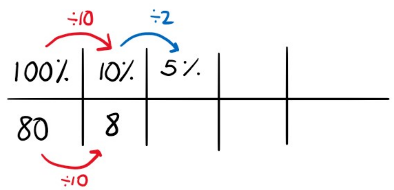


Stage 3 Semi-Abstract

Students should represent the initial amount at 100% in a proportion table.

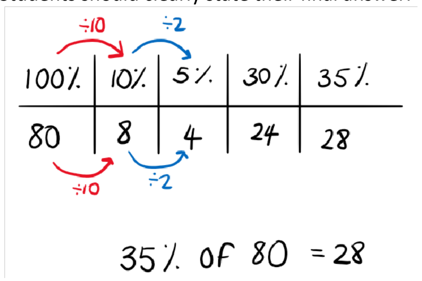


Students should then calculate a suitable percentage(s) to use to calculate the final amount, (10% and 5% in this case).



Students should complete their proportion table to obtain the required final percentage.

Students should clearly state their final answer.



Stage 4 Abstract

Students should understand that 'of' means multiply and determine the multiplier.

$$0.35 \times 80 = 28$$

ONE QUANTITY AS A PERCENTAGE OF ANOTHER

GENERAL TEACHING & LEARNING POINTS	•		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should be able to represent the problem using a bar model.	Students should use a written method using equivalent fractions.	Students should understand that they can write their calculation as a single line of working, which could be entered into a calculator if permitted.
Example(s)	**Example(s) - These should be seen in books**		

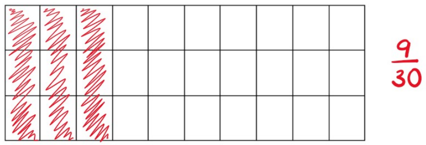
Zuzanna scored 9 out of 30 on a maths test. Work out her score as a percentage.

Stage 1

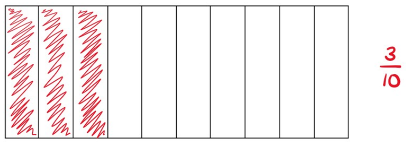
This may be added at a later date.

Stage 2

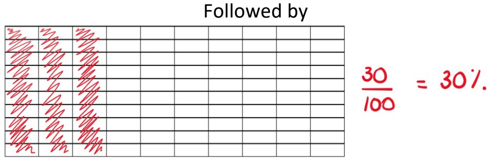
Students should represent the problem as a diagram. They should divide the total amount into an appropriate amount, usually a factor of 100.



Students should understand that percentage means 'per 100'. They should then split their diagram into 100 parts. It may be appropriate to split their diagram using two stages.



Followed by



Stage 3

Students should use their understanding obtained in the Pictorial Stage and the idea that this relates back to equivalent fractions, to show their working as below:

$$\frac{9}{30} \overset{\div 3}{=} \frac{3}{10} \overset{\times 10}{=} \frac{30}{100} = 30\%$$

Stage 4

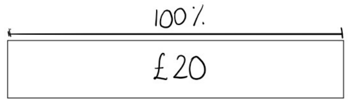
$$\frac{9}{30} \times 100 = 30\%$$

PERCENTAGE INCREASE/DECREASE

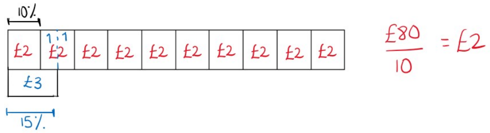
GENERAL TEACHING & LEARNING POINTS	• Students should understand that increasing (or decreasing) by a percentage is equivalent to finding a percentage of an amount above (or below) 100%.		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should be able to represent the problem using a bar model.	Students should use a proportion table to find the required percentage.	Students should be able to use a calculator, through a multiplier, to determine the final amount.
Example(s)	**Example(s) - These should be seen in books**		
Increase :20 by 15%			

This may be added at a later date.

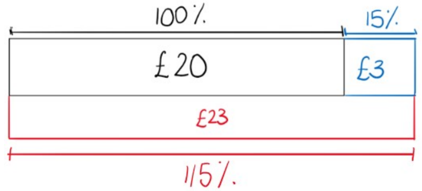
Students should use a bar model to represent the initial 100%.



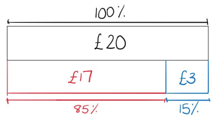
Students should then use the method shown in Finding Percentage of an Amount, to find the percentage to increase (or decrease) by.



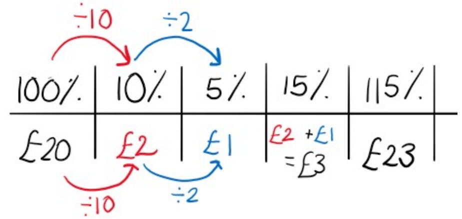
Students should then add on the £3 to the end of their diagram, clearly labelling this as 115%.



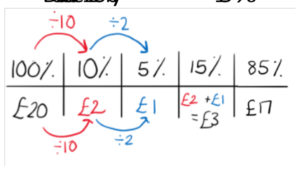
Decrease £20 by 15%



Students should use their understanding of proportionality to set their working out in a proportion table, as shown below.



Decrease £20 by 15%



Students should understand that 'of' means multiply and determine the multiplier.

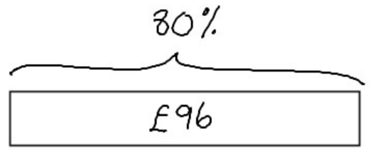
$$1.15 \times 20 = £23$$

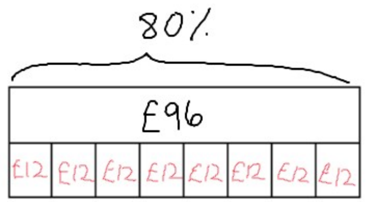
Decrease £20 by 15%

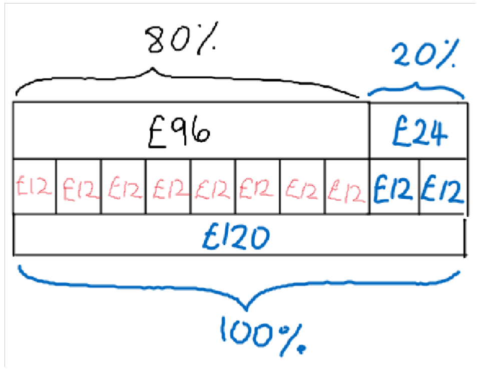
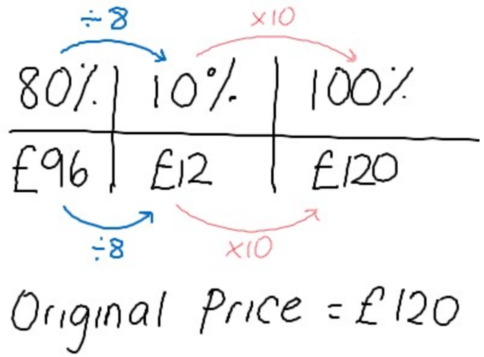
$$0.85 \times 20 = £17$$

REVERSE PERCENTAGES

GENERAL TEACHING & LEARNING POINTS	• Students should understand that increasing (or decreasing) by a percentage is equivalent to finding a percentage of an amount above (or below) 100%.		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should be able to represent the problem using a bar model.	Students should use a proportion table to find the required percentage.	Students should be able to use a calculator, through a multiplier, to determine the final amount.
Example(s)	**Example(s) - These should be seen in books**		
A shop has a 20% off sale. A pair of trainers have a sale price of £96. How much were the trainers before the sale?			

<table>
<tr>
<td>This may be added at a later date.</td>
<td>Students should use a bar model to represent the discounted amount.


Students should then use the method shown in Finding Percentage of an Amount, to find the value of 100%.



</td>
<td>Students should use their understanding of proportionality to set their working out in a proportion table, as shown below.

</td>
<td>80% of Original Amount = £96
$0.8 \times$ Original Amount = £96
Original Amount = $\frac{£96}{0.8}$
= £120
(The first line of working could be omitted here)</td>
</tr>
</table>

Curriculum Area: ALGEBRA

Forming Expressions
Substitution
Simplifying Expressions
Solving Linear Equations
Linear Sequences
Laws of Indices
Expanding Single Brackets
Factorising Single Brackets
Linear Inequalities
Linear and Quadratic Graphs

FORMING EXPRESSIONS

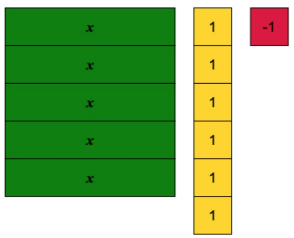
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent a problem using manipulative (cuisenaire rods or algebra tiles).	Students should draw the manipulatives as a diagram in their books, clearly collecting like terms and identifying the zero-pairs.	Students should use a diagram and identify the zero-pairs and cross them out.	Students should use algebra only to be able to collect like terms to find the total.
Example(s)	**Example(s) - These should be seen in books**		

Andy is years' old. Bella is two years' older than Andy. Colin is one year younger than Andy. Danni is twice as old as Bella.
Write an expression for the **total** age of all four people.

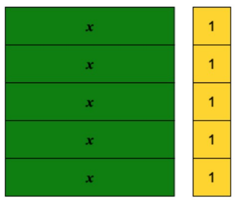
Stage 1 Physical

Represent each person's age using cuisenaire rods, place each person above each other to show comparisons.

Students should collect like terms (see Collecting Like Terms section)



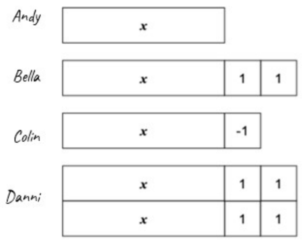
Students should then use the idea of **zero-pairs** to simplify



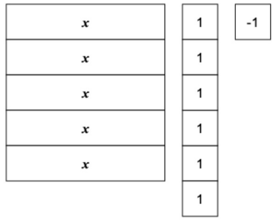
Students should understand that this represents

Stage 2 Pictorial

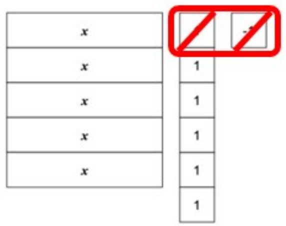
Represent each person's age using cuisenaire rods, place each person above each other to show comparisons.



Students should collect like terms (see Collecting Like Terms section)
Students should then use the idea of



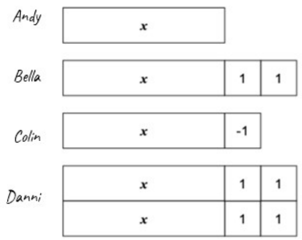
zero-pairs to simplify. Students should circle, label and cross out the zero-pairs



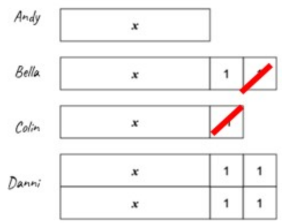
Students should then be able to count the 's and the 1's and that this represents

Stage 3 Semi-Abstract

Represent each person's age using cuisenaire rods, place each person above each other to show comparisons.



Students should then use the idea of **zero-pairs** to simplify, without the need to show the like terms together. Students should circle, label and cross out the zero-pairs.



Students should then be able to count the remaining 's and 1's and that this represents

Stage 4 Abstract

Students should represent each person's age as an algebraic expression.

A x

B x+2

C x-1

D 2x+4   This could be represented as 2(x+2) depending on student's ability

Students should then represent this as one expression clearly showing each expression either added or subtracted.

x + (x+2) + (x-1) + (2x+4)

Notice that each expression is in brackets to ensure that students do not make sign errors with negatives.

SUBSTITUTION INTO ALGEBRAIC EXPRESSIONS

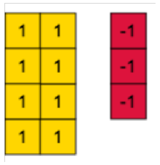
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to algebra tiles to represent the expression in the problem, then replace the variable tiles with numerical tiles.	Students should draw the manipulatives as a diagram in their books, clearly replacing the variable with 1 or -1 tiles.	Students now be able to use the initial diagram and replace the variables with a value.	Students should now be able to apply their understanding of algebraic notation to substitution. At this stage, students will not need a diagram.
Example(s)	**Example(s) - These should be seen in books**		

If x =3 find the value of the expression $4x-3$

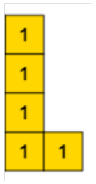
Stage 1 Physical

Using Algebra Tiles to represent the expression

Students should then replace x tiles with 3 1 tiles.



Students should then use the idea of **zero-pairs** to simplify



Students can then count the tiles to determine the answer.

5

Stage 2 Pictorial

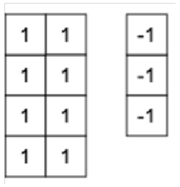
Students should draw the algebra tiles representation in their books

$4x - 3 =$

Students should clearly state (through a diagram) the value of each variable:

Students should then redraw their diagrams replacing the x bars with 3 1 tiles.

Where possible, students should be encouraged to make the number of 1 tiles the same size as the x tile it is replacing.



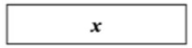
Students should then use the idea of **zero-pairs** to simplify. Students should circle, label and cross out the zero-pairs.

Stage 3 Semi-Abstract

Students should draw the algebra tiles representation in their books

$4x - 3 =$

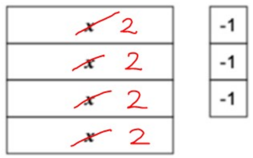
Students should clearly state (through a diagram) the value of each variable, using a number rather than 1s and -1s:



$= 3$

Students should then replace the x tiles by crossing out the x in their diagram and replacing it with the value of x.

$4x - 3 =$



Students should then calculate the value of the expression:

$2 + 2 + 2 + 2 - 3$

$= 4 \text{ x } 2 - 3$

= **5**

Stage 4 Abstract

Students should be able to re-write the algebraic expression with clear algebraic understanding

$4x-3$

$=4\times x-3$

$=4\times 2-3$

$=8-3$

= **5**

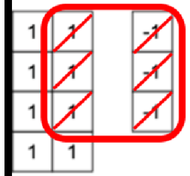
Higher attaining students should be discouraged from using the multiplication sign as above and instead should be encouraged to use brackets:

$4x-3$

$=4(2)-3$

$=8-3$

= **5**

	Students can then count the tiles to determine the answer.  **5**		

COLLECTING LIKE TERMS

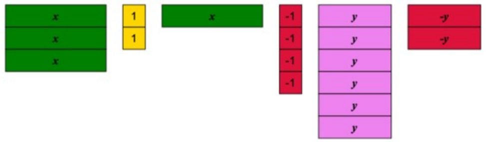
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to algebra tiles to represent the expression in the problem, then collect like tiles and apply the idea of zero-pairs	Students should draw the manipulatives as a diagram in their books, clearly collecting like terms and identifying the zero-pairs.	Students now be able to use the initial diagram and replace the variables with a value.	Students should now be able to apply their understanding of like terms without the need for any diagram.
Example(s)	**Example(s) - These should be seen in books**		

Simplify $3x+2+x-4+6y-2y$

Stage 1 Physical

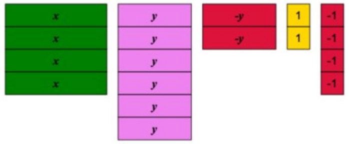
Using Algebra Tiles to represent the expression

Students should then physically move

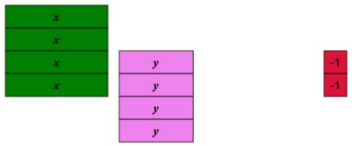


the

tiles so that all identical tiles are together.



Students should use the idea of zero-pairs to simplify the expression.

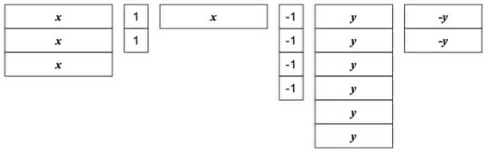


Students should then identify the simplification from the representation.

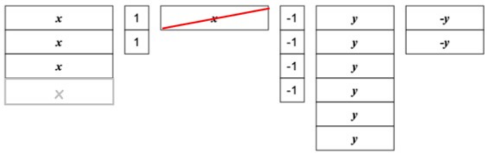
$$4x+4y-2$$

Stage 2 Pictorial

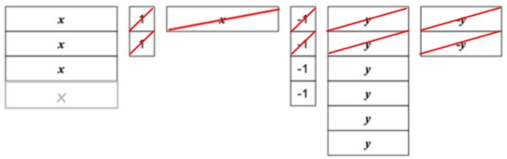
Students should draw the Algebra Tiles in their books, clearly labelling each tile.



Students should then collect like terms cross out a term and adding it to the remaining tiles of the same like term.



Students should then identify and cross through terms that make zero-pairs.

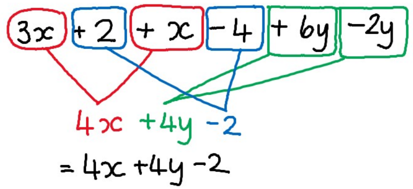


Students should then be able to identify the simplification from this final representation.

$$4x+4y-2$$

Stage 3 Semi-Abstract

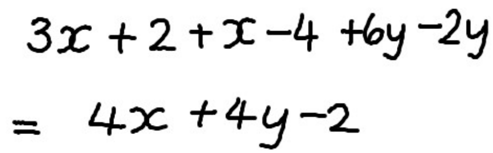
Students should first identify, circle and join like terms.
By this stage students should understand the term includes the sign in front of the algebra part.



Students should be able to clearly justify why the terms $4x$, -2 and $+4y$ do not simplify.

Stage 4 Abstract

Students should be able to collect like terms without the need to circle the like terms.

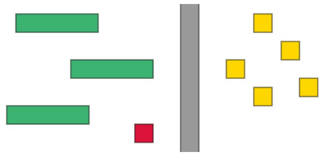


SOLVING LINEAR EQUATIONS

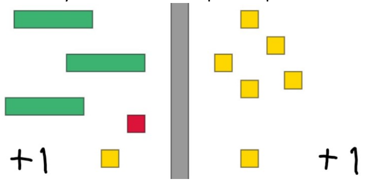
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent a problem using manipulative algebra tiles.	Students should draw the manipulatives as a diagram in their books.	Students should be able to relate the diagrams of the algebra tiles to the equations.	Students should be able to use a formal balancing method to solve the equation.
Example(s)	**Example(s) - These should be seen in books**		
Solve the equation $3x-1=5$			

Students should use Algebra Tiles to represent the equation.

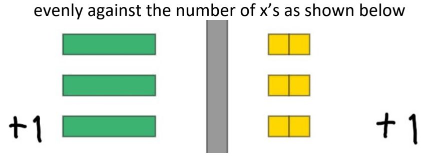
Students should lay tiles on a **mini-whiteboard** so that students can write down their process.
Students should then use the idea of zero-pairs to eliminate ones on one side.



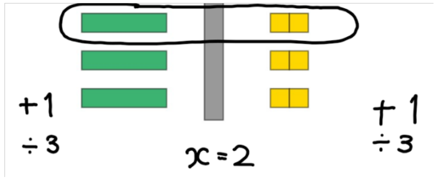
Students should note down on each side of the equation what process they have done to keep the equation balanced.



Students should then remove any zero-pairs and arrange the ones evenly against the number of x's as shown below

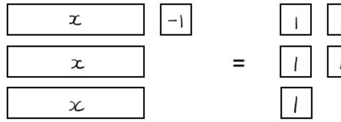


Students should then determine how to find the value of a single x, write this process below any previous operations, and circle this



Students should draw the algebra tiles in their books.

Students should draw the x tiles first and then work in columns to add the ones, as shown below:

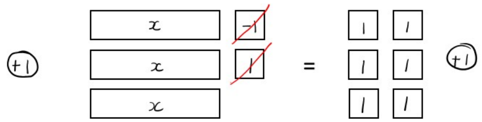


Students could use highlighters to make drawing the tiles easier and quicker.

Students should then use zero-pairs in order to eliminate the 1 or -1 tiles from one side of the equation.

Students should note down what they have added to both sides of the equation in order to do this.

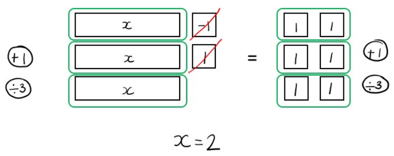
Students should then cross out any zero-pairs.



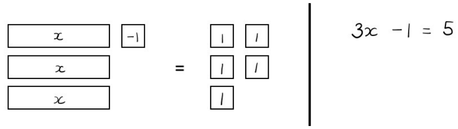
If students have lined the x and 1s in columns systematically then they will be able to easily identify the value of 1 x tile.

Students should clearly write down that they have divided/multiplied below the last operation.

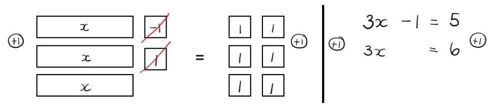
Some students may find it useful to circle the groups which give the value of one x.



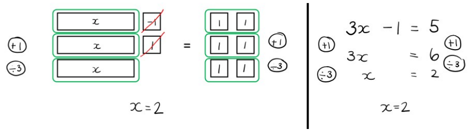
Students should represent the equation in the same way as in the pictorial stage, but with the addition of a formal balancing method also shown.



Students should then form zero-pairs as in the pictorial stage. Students should also include what they have done in the balancing method.



Students should then use a similar diagram to the one used in the pictorial stage.



The student's final solution should be clearly written underneath any diagram or working.

At this stage students should be comfortable with solving equations without the need for a diagram and can solely rely on a formal balancing method.

Students however, at least initially, should show the operation that they have used in each stage of working.

$$3x - 1 = 5$$
(+1) $$3x = 6$$ (+1)
(÷3) $$x = 2$$ (÷3)

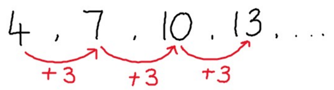
$$x = 2$$

LINEAR SEQUENCES

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent a problem using manipulative, e.g. Cuisenaire Rods.	Students should draw the manipulatives as a diagram in their books.	Students should be able to identify the term-toterm rule and comparing this to the relevant times table.	Students should be able to determine the nth term rule by mentally comparing the sequence to the relevant times table.
Example(s)	**Example(s) - These should be seen in books**		
Work out the nth term rule for the sequence 4, 7, 10, 13, …			

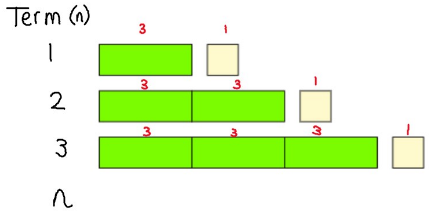
It will be beneficial if students use the manipulatives on top of a whiteboard for this stage.

Students should identify that the sequence increases by 3 each time.

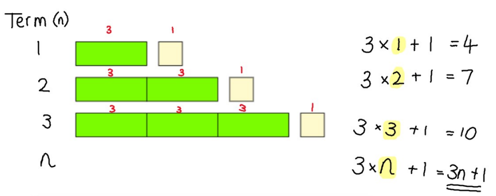


Students should then know that the sequence is linked to the 3 times table.

Students should then represent this using the manipulative. Since the sequence increases by 3, the 3 block will be needed.

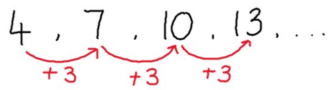


Notice the term is also included and the value of each block is written above.

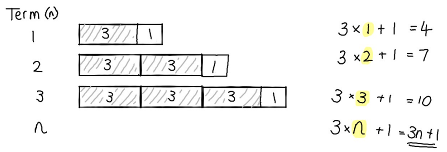


Students should then write each term as a multiple of the term-to-term rule, as shown on the right above.

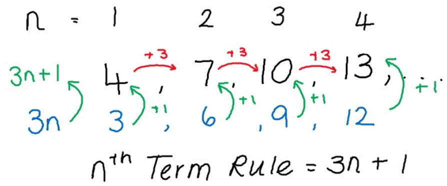
Students should identify that the sequence increases by 3 each time.



Students should then know that the sequence is linked to the 3 times table.



It may be useful for weaker students to shade in the term-to-term blocks to make them easier to count and distinguish.



Students should first write in the term numbers above the sequence.

Students should then determine the term-to-term rule, shown in **red** above.

Students should then know that the sequence is linked to the 3 times table and therefore the sequence 3n is needed. This should be written below, shown in **blue** above.

Students should then determine how they obtain the sequence from the 3n row, this is shown in **green** above. This can be done either as shown above or by subtracting (this may be useful for quadratics).

Students may be able to determine the nth term rule immediately at this stage and state the answer of 3n+1.

Students however MUST be able to clearly justify why by explaining that is the sequence is "one more than the three times table, therefore the nth term rule is 3n+1."

LAWS OF INDICES

GENERAL TEACHING & LEARNING POINTS	**The following apply to the multiplication, division and power on power rule:** • Teachers should ensure thorough understanding through writing out the meaning of each term as repeated multiplication at first instance. • When confident through using this repeated multiplication, only then should students be shown the rule

Multiplication Law

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	N/A	Students should understand the idea of an index and write out the meaning of each term.	Students should now be able to apply their understanding of like terms without the need to write out any terms.
Example(s)		**Example(s) - These should be seen in books**	
Write $4a \times 3a^2$ as a single power of a.			

NOT APPLICABLE	It may be useful to use colour to differentiate between the terms in the question. Students at this stage should clearly write out what each term means as a repeated multiplication. $4a \times 3a^2$ $= 4 \times a \times 3 \times a \times a$ Students should then understand that multiplication is **associative** and therefore the order of multiplication is irrelevant. $4a \times 3a^2$ $= 4 \times a \times 3 \times a \times a$ $= 4 \times 3 \times a \times a \times a$ Students should then evaluate $4a \times 3a^2$ $= 4 \times a \times 3 \times a \times a$ $= 4 \times 3 \times a \times a \times a$ $= 12 \times a^3$ Then leading to a final answer, **<u>without a multiplication sign</u>**. $4a \times 3a^2$ $= 4 \times a \times 3 \times a \times a$ $= 4 \times 3 \times a \times a \times a$ $= 12 \times a^3$ $= 12a^3$	Only when confident with the previous layout, students should be able to write their working as: $4a \times 3a^2$ $= 4 \times 3 \times a^{1+2}$ $= 12a^3$	Students should be able to state the final answer to the question without the need for any working. $4a \times 3a^2$ $= 12a^3$

Division Law

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	N/A	Students should understand the idea of an index and write out the meaning of each term.	Students should now be able to apply their understanding of like terms without the need to write out any terms.
Example(s)		**Example(s) - These should be seen in books**	

Write $12b^5 \div 2b^3$ as a single power of b.

Stage 1 & 2	Stage 3 (part 1)	Stage 3 (part 2)	Stage 4
NOT APPLICABLE	Students should be encouraged to write the original problem as a fraction, and then split this fraction into two parts $12b^5 \div 2b^3$ $= \frac{12 \times b^5}{2 \times b^3}$ $= \frac{12}{2} \times \frac{b^5}{b^3}$ Students at this stage should then write the full meaning of each index $= \frac{12}{2} \times \frac{b \times b \times b \times b \times b}{b \times b \times b}$ Students should then simplify the fractions by cancelling/ $= \frac{\cancel{12}^{6}}{\cancel{2}_{1}} \times \frac{b \times b \times \cancel{b} \times \cancel{b} \times \cancel{b}}{\cancel{b} \times \cancel{b} \times \cancel{b}}$ Students should then state their final answer $= \frac{6}{1} \times b^2$ $= 6b^2$	Students should begin the second part of this stage in the same way as previous. $12b^5 \div 2b^3$ $= \frac{12 \times b^5}{2 \times b^3}$ $= \frac{12}{2} \times \frac{b^5}{b^3}$ Students should then apply the law of index explicitly. $= 6 \times b^{5-3}$ $= 6b^2$	Students should be able to state the final answer to the question without the need for any working. $12b^5 \div 2b^3$ $= 6b^2$

Power-on-Power Law

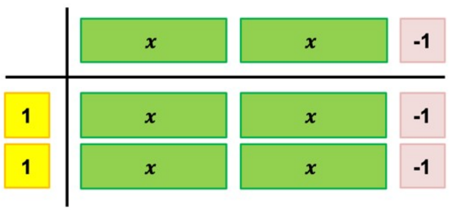
<table>
<tr><th>Stage 1 Physical</th><th>Stage 2 Pictorial</th><th colspan="2">Stage 3 Semi-Abstract</th><th>Stage 4 Abstract</th></tr>
<tr><td>N/A</td><td>N/A</td><td colspan="2">Students should understand the idea of an index and write out the meaning of each term.</td><td>Students should now be able to apply their understanding of like terms without the need to write out any terms.</td></tr>
<tr><td colspan="2">Example(s)</td><td colspan="3">Example(s) - These should be seen in books</td></tr>
<tr><td colspan="5">Write $(2b^4)^3$ as a single power of b.</td></tr>
<tr><td>NOT APPLICABLE</td><td colspan="2">$(4x^2)^3$
$= 4x^2 \times 4x^2 \times 4x^2$
$= 4 \times 4 \times 4 \times x \times x \times x \times x \times x \times x$
$= 64 \times x^6$
$= 64x^6$</td><td>$(4x^2)^3$
$= 4^3 \times (x^2)^3$
$= 64 \times x^{2 \times 3}$
$= 64x^6$</td><td>Students should be able to state the final answer to the question without the need for any working.
$(4x^2)^3$
$= 64x^6$</td></tr>
</table>

EXPANDING SINGLE BRACKETS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to use **algebra tiles** to represent an area model.	Students should be able to link expanding brackets to the areas of rectangles. This should be represented in an adapted grid method that illustrates different variables as different sizes.	Students should extend their knowledge in the Concrete stage to remove the idea of lengths whilst still associating the working to area if required	Students should not need to use a grid, students at this stage should understand that each term inside the bracket must be multiplied by the term outside.
Example(s)	**Example(s) - These should be seen in book**		

Expand 2(2x – 1)

Stage 1



Students should understand that means "two lots of ".

Students should then see that there are 4 's and 2 's, so

Stage 2

x	2x	-1
2	Area = 4x	Area = - 2

Students should then understand that the expansion is by looking inside the grid

Notice that the sign is always included even if positive to emphasise the multiplication of a positive or negative number.

Stage 3

Students should then be able to expand a single bracket without the need to associate the size of each segment to the value.

x	2x	−1
2	4x	−2

$= 4x - 2$

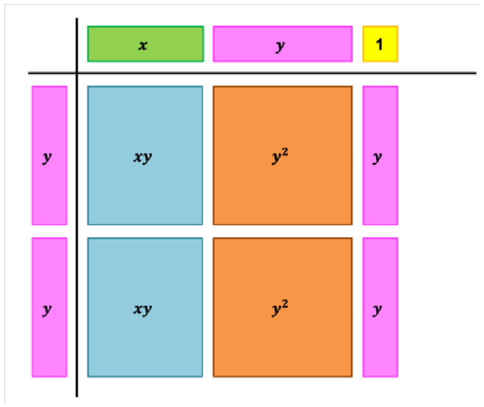
Stage 4

$2(2x - 1)$

$= 4x - 2$

Expand 2y(x + y + 1)

Stage 1



Stage 2

X	x	+y	+1
2y	Area = 2xy	Area = $+2y^2$	Area= +2y

$= 2xy + 2y^2 + 2y$

Stage 3

x	x	+y	+1
2y	2xy	$+2y^2$	+2y

$= 2xy + 2y^2 + 2y$

Stage 4

$2y(x + y + 1)$

$= 2xy + 2y^2 + 2y$

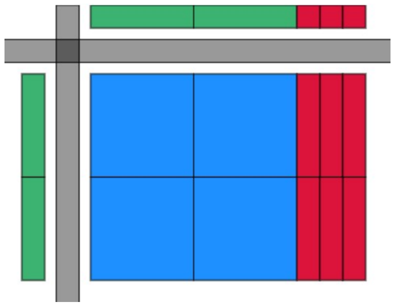
FACTORISING SINGLE BRACKETS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to use **algebra tiles** to represent an area model.	Students should be able to link factorising to the areas of rectangles. This should be represented in an adapted grid method that illustrates different variables as different sizes.	Students should extend their knowledge in the Concrete stage to remove the idea of lengths whilst still associating the working to area if required	Students should not need to use a grid, students at this stage should be able to determine the HCF of the terms and determine the final answer without working.
Example(s)	**Example(s) - These should be seen in books**		
Factorise: $4x^2 - 6x$			

Students should arrange Algebra tiles in such a way that they form a perfect rectangle.

Students should then understand from expanding that the divisor goes outside of the bracket (the height of the diagram) and the quotient goes inside of the bracket (the width of the diagram.

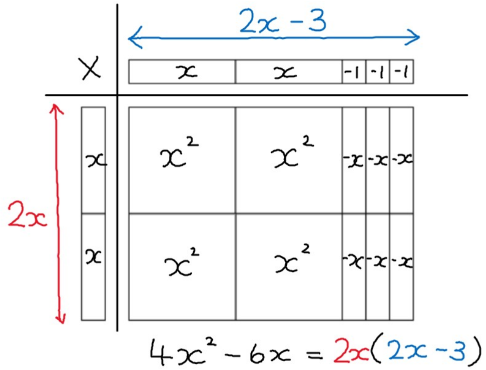
Students should use additional tiles to identify this.
It may be useful for students to draw a line along the left side and top to help distinguish the tiles.



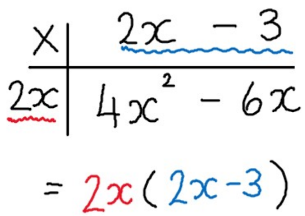
Students should then be able to state $4x^{\%} - 6x = 2x(2x - 3)$

Students should draw the tiles in their books, using the same ideas as the Physical Step.

At this stage their sizes should be the same as the physical tiles.



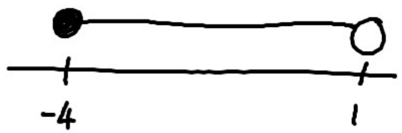
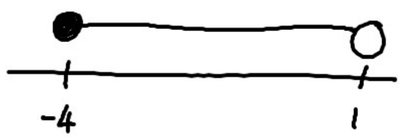
Students should represent the expression in a grid, but without the need for it to be a scaled diagram.



Students should be able to state their factorised answer without the need to use a diagram.

$$4x^2 - 6x$$
$$= 2x(2x - 3)$$

SOLVING INEQUALITIES

GENERAL TEACHING & LEARNING POINTS	• TBA		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	N/A	Students should split the inequality into two parts, solving each separately.	Students should be able to apply the same operation to all three parts of the inequality, therefore removing the need to split the inequality into two parts.
Example(s)	**Example(s) - These should be seen in books**		
Solve the inequality $-3 \le 2x+5 < 7$. Represent your solution on a number line.			
NOT APPLICABLE	**NOT APPLICABLE**	Students should use their solving equations skills to be able to perform similar operations. NOTE: The inequality symbols should be left in throughout and not replaced with an = sign. $-3 \le 2x+5 < 7$ $-3 \le 2x+5$ (−5) → $-8 \le 2x$ (÷2) → $-4 \le x$ $2x+5 < 7$ (−5) → $2x < 2$ (÷2) → $x < 1$ Students should then combine their answers to form the final solution set $-4 \le x < 1$ Students should then represent their answer on a number line as specified in the question. 	At the Abstract Stage, students should not need to separate the two parts of the inequality, instead they should apply the inverse operations to all three parts of the inequality equation. $-3 \le 2x+5 < 7$ (−5) $-8 \le 2x < 2$ (÷2) $-4 \le x < 1$ Students should then represent their answer on a number line as specified in the question. 

CALCULATING THE GRADIENT OF A STRAIGHT LINE

GENERAL TEACHING & LEARNING POINTS	• TBA		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should be able to calculate the gradient from a graph.	Students should be able to calculate the gradient from two points and drawing a sketch.	Students should be able to calculate the gradient from two points and drawing a sketch.

Example(s)

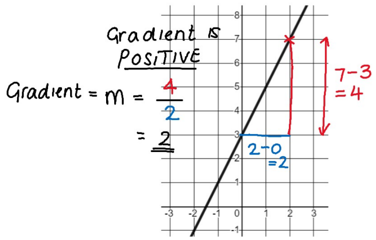
Example(s) - These should be seen in book s

NOT APPLICABLE

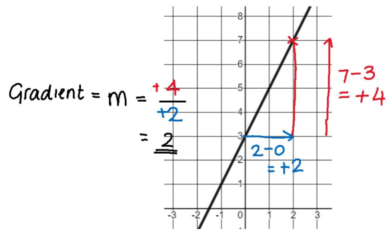
Stage 2 Pictorial

Determine the gradient of the straight line below

At this stage students work out if the gradient is positive or negative, work out the absolute value of the gradient and add in the – sign afterwards if needed.



Higher attaining students will be able reduce this to:

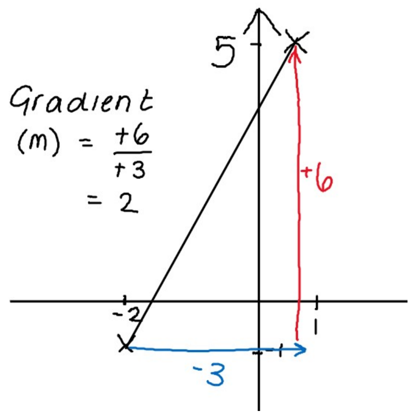


Stage 3 Semi-Abstract and Stage 4 Abstract

Calculate the gradient of the line that passes through (-2,-1) and (1,5)

At this stage the problem has been prepresented as a diagram, similar to that of the Pictorial Stage.

This has been represented on an axis, however, this is not necessary. A simple sketch would suffice.



At this stage students should use the $\frac{Change\ in\ y}{Change\ in\ x} =$ 0;20; 1;21" formula.

$$(-2, -1), (1, 5)$$

$$x_1 \quad y_1 \qquad x_2 \quad y_2$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5 - {}^{-}1}{1 - {}^{-}2}$$

$$= \frac{6}{3}$$

$$= 2$$

PLOTTING LINEAR GRAPHS

GENERAL TEACHING & LEARNING POINTS	• TBA		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should draw an accurate graph on an axis.	Students should draw an accurate graph on an axis.	At this stage students should be able to sketch the graph, indicating any intersection points with the axes.
Example(s)	**Example(s) - These should be seen in book s**		
	Plot the graph of $y = 4x - 1$		

NOT APPLICABLE

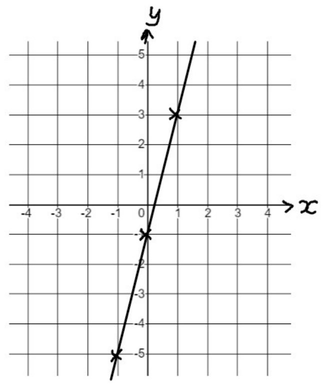
Students should form a table of values.

Students should use substitution only in order to find each co-ordinate.

$y = 4x - 1$

x	-2	-1	0	1	2
y	-9	-5	-1	3	7

(-2,-9) (-1,-5) (0,-1) (1,3) (2,7)



Students should use a table of values, however, starting at the x=0 term.

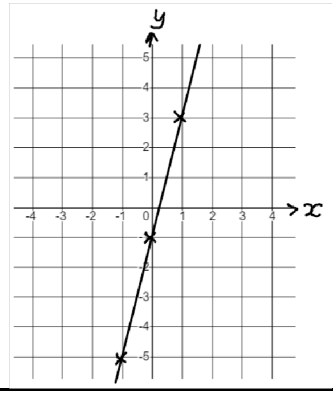
Students should then understand that the gradient is 4 and therefore for each the y value will increase by 4 for each increase of 1 in x.

$y = 4x - 1$

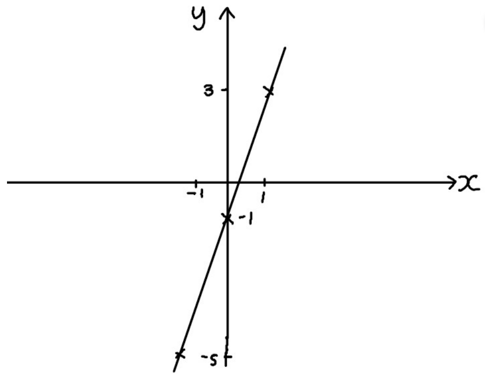
x	-2	-1	0	1	2
y	-9	-5	-1	3	7

+4 +4 +4 +4

At
this stage students should not need to explicitly write each co-ordinate to be plotted.



Students should first plot the y-intercept, followed by increasing/decreasing by the gradient for every increase in one unit of x.



PLOTTING QUADRATIC GRAPHS

GENERAL TEACHING & LEARNING POINTS	• TBA		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should be able to plot a quadratic curve using a detailed table of values.	Students should be able to plot a quadratic curve using a two-row table of values.	Students should be able to plot a quadratic using full algebraic methods, including completing the square and factorising and solving.
Example(s)	**Example(s) - These should be seen in book s**		

Plot the graph of $y = x^{\%} + 2x - 3$. State the co-ordinates of any poi nts of intersection with the axes.

Stage 1:

NOT APPLICABLE

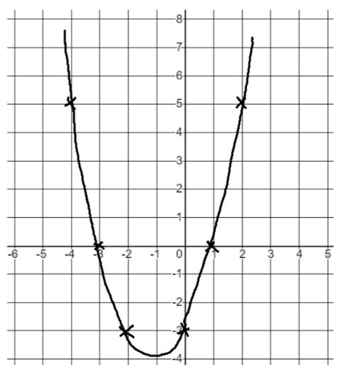
Stage 2:

Students should form a table of values where each term in the equation is separate

Students should then add each part to form a coordinate.

$y = x^2 + 2x - 3$

x	-4	-3	-2	-1	0	1	2
x^2	16	9	4	1	0	1	4
$+2x$	-8	-6	-4	-2	0	+2	+4
-3	-3	-3	-3	-3	-3	-3	-3
y	5	0	-3	-2	-3	0	5

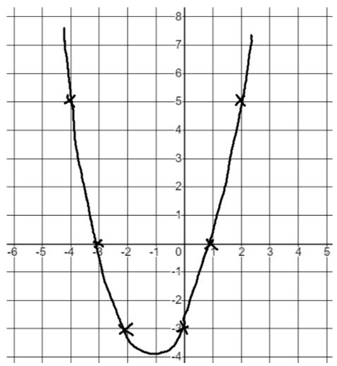


Stage 3:

Students will be able to form a table of values with just x and y rows.

x	-4	-3	-2	-1	0	1	2
y	5	0	-3	-2	-3	0	5

Students will then be able to plot the graph from this.



Stage 4:

Students should be able to sketch the graph by using a combination of factorising to determine the roots, completing the square and intercepts.

$y = x^2 + 2x - 3$ → y-intercept

Finding Roots:

$x^2 + 2x - 3 = 0$

$(x - 1)(x + 3) = 0$

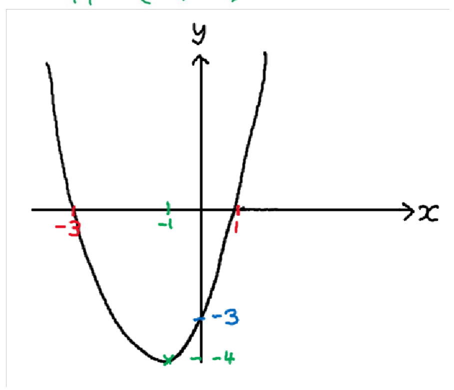
$x = 1 \quad x = -3$

Finding Turning Point

$(x + 1)^2 - 1 - 3$

$= (x + 1)^2 - 4$

$TP = (-1, -4)$



Curriculum Area:

RATIO AND PROPORTION

Converting Units
Writing Ratios as Fractions
Sharing in a Ratio
Compound Measures
Direct Proportion
Best Buy Problems Exchange
Rates

CONVERTING UNITS

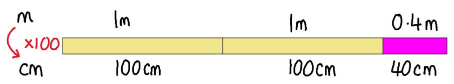
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students could use Cuisenaire Rods (base 10) to represent the original length.	Students should use a bar model to represent a similar idea to that of the Physical Stage.	Students should use a proportion table to scale up or down each quantity.	Students should be able to state the final conversion without any working.
Example(s)	**Example(s) - These should be seen in books**		

Convert 2.4m into cm.

Stage 1 Physical

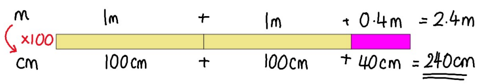
It will be beneficial if students use the manipulatives on top of a whiteboard for this stage.

Students should represent the 2.4m using two '10' rods and one '4' rod.



Notice the scale factor has been indicated on the left hand side.

Students should then sum above and below.

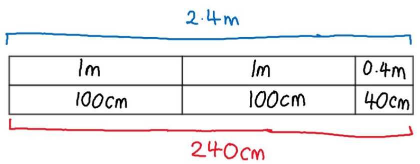


Students should then state their final answer.

2.4m = 240cm

Stage 2 Pictorial

Students should represent the problem as a bar model clearly equating each part of the bar model in both units, as shown below.



Notice, the diagram is roughly to scale (0.4m is smaller than the 1m), this emphasises that, when converted, the 0.4m cannot be more than 100cm.

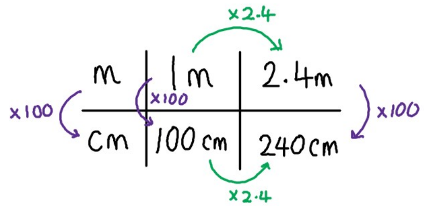
Students should then state their final answer.

2.4m = 240cm

Stage 3 Semi-Abstract

Students should set the work out in a proportion table.

ALL students should be able to perform the conversion in two different ways, shown in green and purple below.



Students should then state their final answer.

2.4m = 240cm

Stage 4 Abstract

Students will be able to state the conversion immediately.

2.4m = 240cm

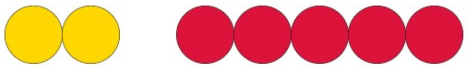
WRITING RATIOS AS A FRACTIONS

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent a problem using counters.	Students should be able to represent the information using a bar model.	Students should be able to use a single bar model to aid their understanding	Students should be able to write the fraction without the need for a diagram.
Example(s)	**Example(s) - These should be seen in books**		

The ratio of yellow to red counters is 2:5. What fraction of the counters is yellow?

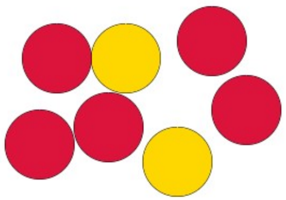
Stage 1 Physical

Students should be able to represent the problem as below.



Notice that this clearly shows the different counters separately.

Students should then physically move the counters together, this is important so that students understand that in total there are 7 parts.



Students should then apply their understanding of what a fraction is to identify that 5 out of the 7 counters are red. Students should clearly write this.

2 out of 7 are yellow

$\frac{2}{7}$ are yellow

Stage 2 Pictorial

Students should represent this as a bar model, with colour side-by-side.

Y	Y

:

R	R	R	R	R

Students should then combine each side into a single bar to represent the total number of parts.

Y	Y	R	R	R	R	R

Students should then add the fraction of each colour underneath their representation.

Y	Y	R	R	R	R	R

$\frac{2}{7}$ $\frac{5}{7}$

$\frac{2}{7}$ are yellow

Stage 3 Semi-Abstract

Students should understand the idea of a ratio by this stage.

Students should therefore be able to draw the diagram below:

Y	Y	R	R	R	R	R

Students should then be able to state the appropriate fraction from this diagram alone.

$\frac{2}{7}$ are yellow

Stage 4 Abstract

Students should be able to represent the problem using a table. Consistency is important here as this layout will be used regularly during sharing in a ratio.

Yellow : Red	Total Parts
2 : 5	2 + 5 = 7

$\frac{2}{7}$ are yellow

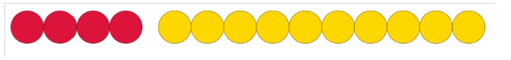
SIMPLIFYING RATIO

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi**
Students should be able to represent a problem using manipulatives (counters or multi-link cubes).	Students should be able to represent the information using a 'comparative' bar model and a 'cumulative' bar model.	Students should use a 'compar abstract knowledge that each p using divis
Example(s)	**Example(s) - These should b**	

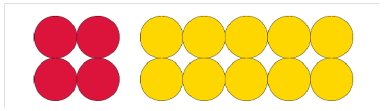
Simplify the ratio 4:10.

Stage 1 Physical

Students should be able to represent the ratio using counters.



Students should then attempt to re-arrange the counters into the same number of rows, two in this case.

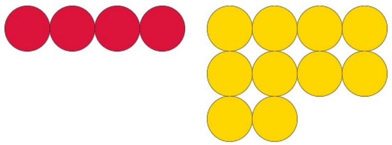


Students should apply their understanding of what a ratio is to remove all rows of counters except the first.

Students should then be able to state that 4:10 can be simplified to 2:5.

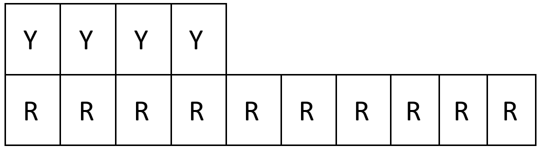
Discussion Point

Students should be able to confidently explain why the counters cannot be arranged as below

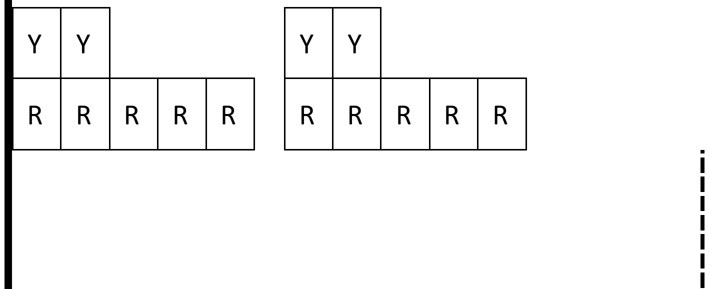


Stage 2 Pictorial

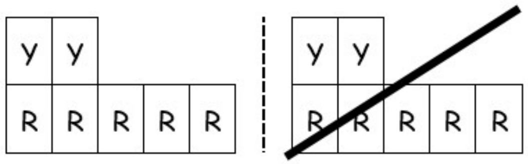
Students should represent this as a 'comparative' bar model, with colour on top of each other



Students should then recognise that the diagram would be equal if two yellow parts were moved as shown:



Students should apply their understanding of what a ratio is to remove all but one group and the ratio remains equal.



R : Y

= 2 : 5

Stage 3 Semi

Students should represent t

Y Y Y Y : R R R

Students should then be able to
the ratio could be g
Higher attaining students cou
highest common facto

Students should circl

Students should then understa
number of each colour is the sa
group

R :

= 2 :

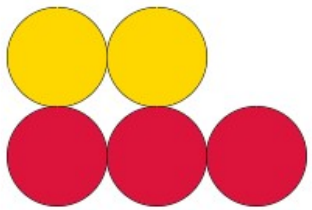
SHARING IN A RATIO – SHARING TOTAL

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent a problem using manipulatives (counters or multi-link cubes).	Students should be able to represent the information using a 'comparative' bar model and a 'cumulative' bar model.	Students should use a 'comparative' bar model and use abstract knowledge that each part is of equal value, thus using division.	Students should be able to share the ratio without needing a diagram. Students can then use equality of ratios and proportions.
Example(s)	**Example(s) - These should be seen in book**		

There are 60 counters in a box. They are either yellow or red. The ratio of yellow counters to red counters is 2:3. How many of each colour are in the box?

Stage 1 Physical

Students should be able to represent the problem as below.



Students should understand that the total value of all five parts is 60 and all parts represent the same amount, i.e. the manipulative below represents 60.

Students should then identify the value of one cube:

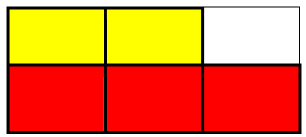
Students should then understand since there are 2 yellow and 3 red blocks this represents:
Yellow: 12 + 12 = 24
Red: 12 + 12 + 12 = 36

[Notice that this is represented as repeated addition and not multiplication]

So there are 24 yellow and 36 red.

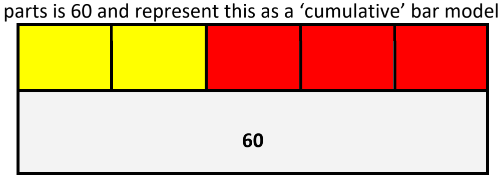
Stage 2 Pictorial

Students should be able to represent the information in the question pictorially as a 'comparative' bar model as below:

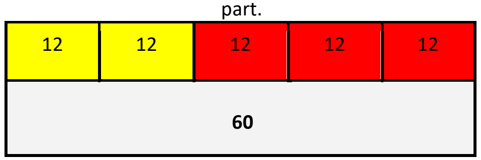


[Notice this is represented as yellow above red, this makes it easier to compare the quantities and also for more difficult problems - see later]

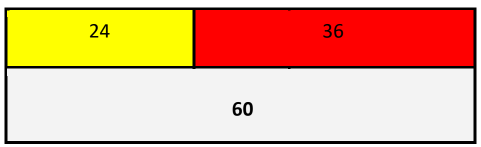
Students should understand that the total value of all five parts is 60 and represent this as a 'cumulative' bar model.



Students should then be able to identify the value of each part.

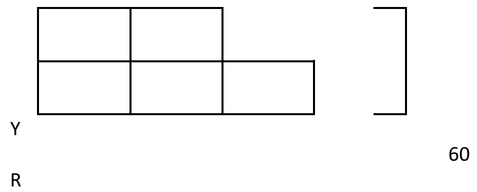


tudents will then see the numbe



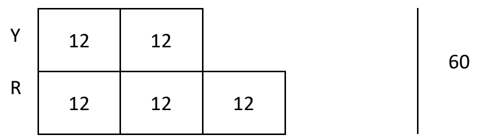
Stage 3 Semi-Abstract

Students should be able to represent the information in the question pictorially, including the total number, as below:

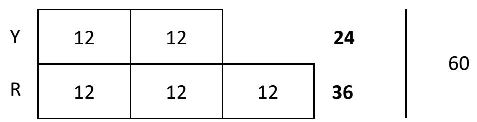


Students should then know to find the value of each part is the total divided by the number of parts.

Students should then annotate the value of each part on their diagram:



Totalling each colour



Stage 4 Abstract

Students should be able to identify the <u>total</u> number of parts, then identify the scale factor to multiply to make the total parts to the total number of counters in this case.

Note: Parts in <u>black</u> are step one, parts in <u>blue</u> are step two.

Yellow : Red Total Parts

2 : 3 5

X12 X12 X12

24 : 36 60

Students should then clearly state the answer either as a ratio or writing the number of each colour in this case. Which of these will depend on the exact wording of each question.

Y:R = **24:60**

OR

Yellow = **24 counters** Red = **36 counters**

	So there are 24 yellow and 36 red.	So there are 24 yellow and 36 red.	

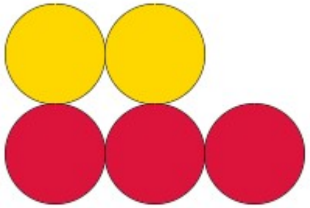
SHARING IN A RATIO - DIFFERENCES

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent a problem using manipulatives (counters or multi-link cubes).	Students should be able to represent the information using a 'comparative' bar model and a 'cumulative' bar model.	Students should use a 'comparative' bar model and use abstract knowledge that each part is of equal value, thus using division.	Students should be able to share the ratio without needing a diagram. Students can then use equality of ratios and proportions.
Example(s)	**Example(s) - These should be seen in books**		

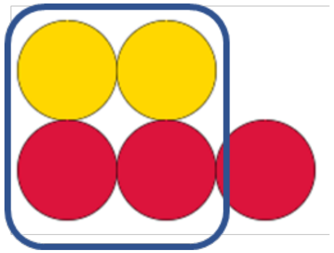
They are either yellow or red counters in a box. The ratio of yellow counters to red counters is 2:3. There are 60 **more** red counters than yellow. How many yellow counters and red counters were in the box?

Stage 1

Students should be able to represent the problem as below.



Students should understand that the four blocks circled have equal parts and the extra red block represents the 60 extra red counters.



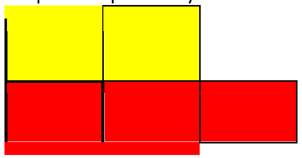
So each block represents 60

Students should then understand since there are 2 yellow and 3 red blocks this represents:
Yellow: 60 + 60 = 120
Red: 60 + 60 + 60 = 180

So there are 120 yellow and 180 red.

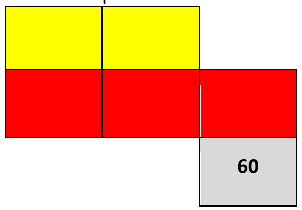
Stage 2

Students should be able to represent the information in the question pictorially as below:

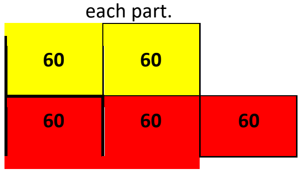


[Notice this is represented as yellow above red, this makes it easier to compare the quantities and also for more difficult problems - see later]

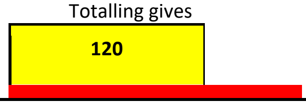
Students should understand that the total value of all five parts is 60 and represent this as a bar model.



Students should then be able to identify the value of each part.

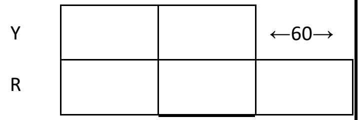


Totalling gives

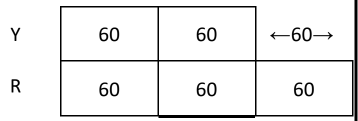


Stage 3

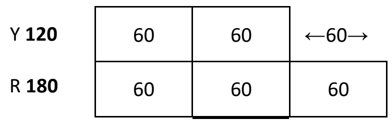
Students should be able to represent the information in the question pictorially, including the difference, as below:



Students should then understand each part must represent the same value.
Students should annotate their diagram.



Students should then be able to total each part.



So there are 120 yellow and 180 red.

Stage 4

Students should be able to identify the difference in the number of parts, then divide to determine the amount represented by 1 part.

Note: Parts in black are step one, parts in blue are step two.

Yellow : Red Parts	Difference
2 : 3	1
X60 X60	X60
120 : 180	60

Students should then clearly state the answer either as a ratio or writing the number of each colour in this case. Which of these will depend on the exact wording of each question.

Y:R = **120:180**

OR

Yellow = **120 counters** Red = **180 counters**

	180 So there are 120 yellow and 180 red.		

SHARING IN A RATIO – GIVEN ONE AMOUNT

Stage 1 **Physical**

Students should be able to represent a problem using manipulatives (counters or multi-link cubes).

Stage 2 **Pictorial**

Students should be able to represent the information using a ‘comparative’ bar model and a ‘cumulative’ bar model.

Stage 3 **Semi-Abstract**

Students should use a ‘comparative’ bar model and use abstract knowledge that each part is of equal value, thus using division.

Stage 4 **Abstract**

Students should be able to share the ratio without needing a diagram. Students can then use equality of ratios and proportions.

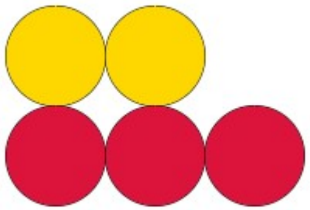
Example(s)

Example(s) - These should be seen in books

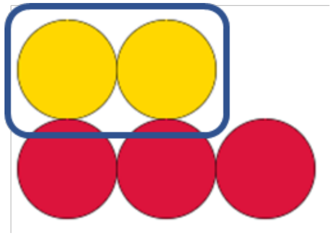
They are either yellow or red counters in a box. The ratio of yellow counters to red counters is 2:3. There are 60 yellow counters. How many counters are in the box in **total**?

Stage 1

Students should be able to represent the problem as below.



Students should understand that the yellow blocks represent 60.



So each block represents 30.

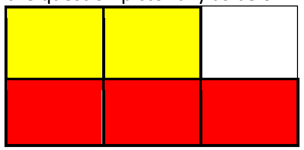
Students should then understand that the 3 red blocks this represents:

Red: 30 + 30 + 30 = 90

So there are 60 + 90 = 150 in total.

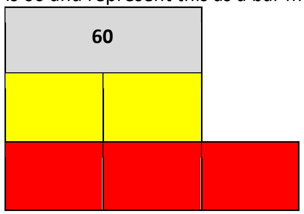
Stage 2

Students should be able to represent the information in the question pictorially as below:

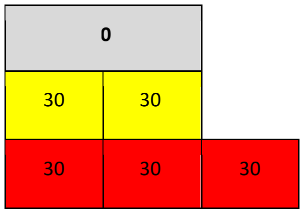


[Notice this is represented as yellow above red, this makes it easier to compare the quantities and also for more difficult problems - see later]

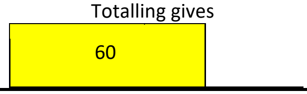
Students should understand that the total value of all five parts is 60 and represent this as a bar model.



Students should then be able to identify the value of each part.

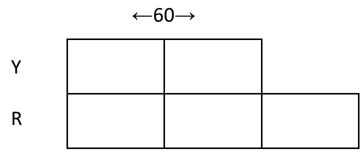


Totalling gives

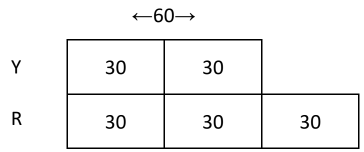


Stage 3

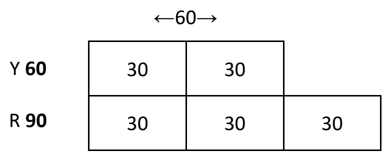
Students should be able to represent the information in the question pictorially, including the total yellow counters., as below:



Students should then understand each part must represent the same value.
Students should annotate their diagram.



Students should then be able to total each part and the overall total.

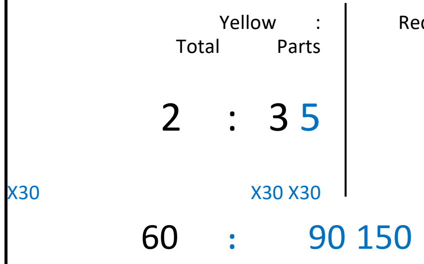


60+90 = 150 counters in total

Stage 4

Students should be able to identify the number of parts represented by the given quantity, then divide to determine the amount represented by 1 part.

Note: Parts in black are step one, parts in blue are step two.



Students should then clearly state the answer either as a ratio or writing the number of each colour in this case. Which of these will depend on the exact wording of each question.

Y:R = 60:90

Total = 60 + 90 = **150**

	90 **60+90 = 150 counters in total**		

COMPOUND MEASURES

GENERAL TEACHING & LEARNING POINTS	
	• Students should have a secure understanding of units and what these units mean. E.g. mph is "how many miles something travels per hour" or Pressure "How many Newtons per every 1 cm^2" • Students should apply their understanding of proportion when working with Compound Measures • Try to avoid using formula triangles with Compound Measures as these do not promote teaching for understanding.

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
N/A	Students should use a bar model to represent the problem.	Students should use a proportion table to scale up or down each quantity.	Students should confidently state and use the Compound Measures formulae without the need for an alternative representation.
Example(s)	**Example(s) - These should be seen in books**		

A car travels at a speed of 45 mph for 20 minutes. How far does it travel in this time?

Stage 1

NOT APPLICABLE

Stage 2

Students should represent this as a bar model.

Notice that the distance travelled in 1 hour is represented using equal length bars.

45 miles
1 hour

Students should then understand that 20 minutes is one third of an hour and therefore this needs to be split into three equal parts.

Students should then use proportion to calculate the distance travelled in this time.

15 miles	15 miles	15 miles
45 miles		
1 hour		
20 mins	20 mins	20 mins

Stage 3

At this stage students should use a proportion table to determine the unknown quantity.

÷3

Distance	45 miles	15 miles
Time	1 hour	$\frac{1}{3}$ hour

÷3

Stage 4

Students should use and substitute into the formula.

Avoid using the formula triangles, however, this should be the only stage at which then could be used if students are unable to understand the basic concepts after significant time has been spent.

Students should clearly state the formula they are using (in its original form) followed by any re-arrangement as a separate stage of working.

Students should state the value of quantities in the question, with particular emphasis on units.

S = 45 mph t = $\frac{1}{3}$ hour

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Distance = Speed x Time

Distance = 45 x $\frac{1}{3}$ = **15 miles**

	Students should then be able to identify that in 20 minutes the car travelled 15 miles.		

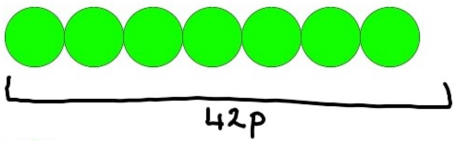
DIRECT PROPORTION

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent this problem using counters or multi-link cubes.	Students should be able to represent the information using a bar model in their books.	Students should use a direct proportion table to be able to represent and solve the problem	Students should be able to use direct proportion using the same format as simplifying a ratio.
Example(s)	**Example(s) - These should be seen in books**		

7 pens cost 42p. Wha : is the cost of 1 pen?

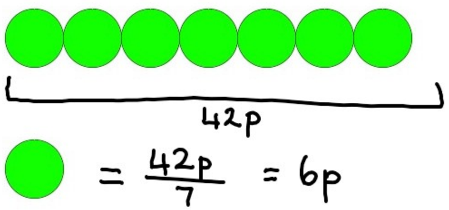
Stage 1 Physical

Students should represent the problem by using 7 counters or cubes to represent the 7 pens.



Students should then be able to work out the value of 1 pen.

Higher attaining students will be able to spot that, for other problems, that it isn't always necessary to find the value of 1 part first. E.g. 8 pens cost 80p work out the cost of 2 pens. Students will be able to divide by 4 in one step rather than dividing by 8 then multiplying by 2.



Students should then clearly answer the question.

1 pen = 6p

Stage 2 Pictorial

Students should draw the diagram below in their books.

1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen
42p						

Students should then clearly show that they have divided the 42p into 7 parts by adding this to their diagram and showing the division calculation.

1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen
42p						
6p	6p	6p	6p	6p	6p	6p

$\frac{42p}{7}$ = 6p

Students could then circle the part of their diagram that gives the value of 1 pen.

1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen	1 Pen
42p						
6p	6p	6p	6p	6p	6p	6p

$\frac{42p}{7}$ = 6p

Students should then clearly answer the question.

Stage 3 Semi-Abstract

At this stage students should use a proportion table to determine the unknown quantity.

÷7

Pens	7 pens	1 pen
Cost	42p	6p

÷7

Students should then clearly answer the question.

1 pen = 6p

Stage 4 Abstract

Students should represent the problem in a table.

Students should clearly show that the division operations need to be applied to both sides to represent the direct proportionality.

Pens : Cost

7 pens : 42p ÷7 1 pen : 6p ÷7

Students should then clearly answer the question.

1 pen = 6p

	1 pen = 6p		

BEST BUY PROBLEMS

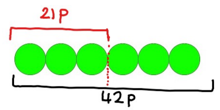
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent this problem using counters or multi-link cubes.	Students should be able to represent the information using a bar model in their books.	Students should use a direct proportion table to be able to represent and solve the problem	Students should be able to use direct proportion using the same format as simplifying a ratio.
Example(s)	**Example(s) - These should be seen in books**		

Shop A sells 6 pens for 42p. Shop B sells 9 pens for 72p.
Bella wants to buy some pens from one of the shops. Which shop should Bella buy th pens from?

Stage 1 Physical

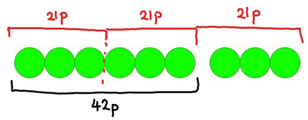
Students should represent the problem by using 7 counters or cubes to represent the 7 pens.

Students should then be able to work out the value of 3 pens.

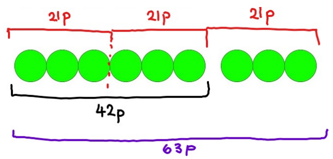


Students could use a unitary approach here, however, students should be encouraged to be efficient with their method and therefore since 9 is a multiple of 3 this would be more effective.

Students should then add in the additional counters to their diagram in order to work out the cost of an equivalent number of pens as Shop B but from Shop A.



Students should then work out the cost from Shop A

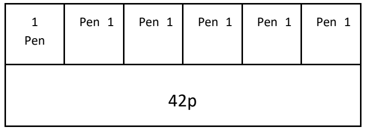


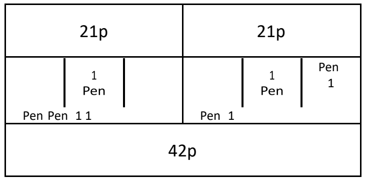
Students should then clearly answer the question.

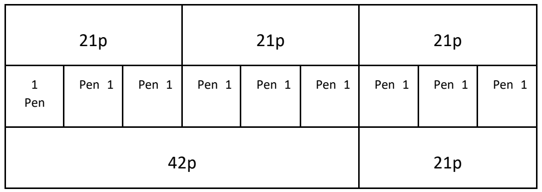
Bella should buy from Shop A

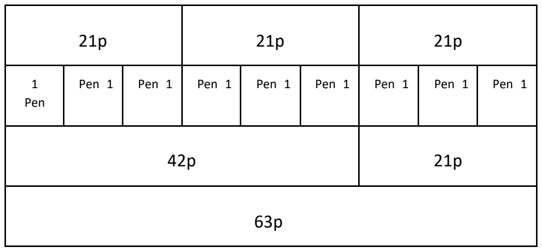
Stage 2 Pictorial

Students should draw similar diagrams to the Physical Stage below in their books, this time using a bar model









Students should then clearly answer the question.

Stage 3 Semi-Abstract

At this stage students should use a proportion table to determine the cost of pens in Shop A.

÷2 x3

Shop A

Pens	6 pens	3 pens	9 pens
Cost	42p	21p	63p

÷2 x3

Bella should buy from Shop A

Stage 4 Abstract

Students should represent the problem in a table.

Students should clearly show that the division operations need to be applied to both sides to represent the direct proportionality.

Shop A

Pens : Cost pens6 : 42p

÷2 3 pens : 21p ÷2

x3 pens9 : 63p x3

Students should then clearly answer the question.

Bella should buy from Shop A

	Bella should buy from Shop A		

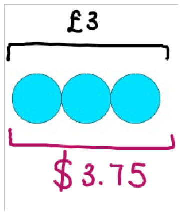
EXCHANGE RATES

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent this problem using counters or multi-link cubes.	Students should be able to represent the information using a bar model in their books.	Students should use a direct proportion table to be able to represent and solve the problem	Students should be able to use direct proportion using the same format as simplifying a ratio.
Example(s)	**E ꞏample(s) - These should be seen in books**		

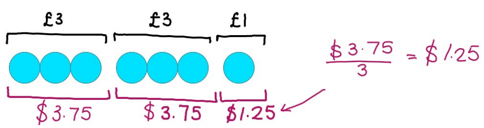
The exchange rate from Pounds (ː) into US Dollars ($) is £3 = $3.75.
Calculate how many Dolla ꞌs ($) are equivalent to £7.

Stage 1 Physical

Students should be able to represent the initial problem and information with a manipulative.



Students should then be able to scale up the given quantity, by either using a unitary method or by understanding that £7 is 2 x £3 + £1, as shown:



If a unitary method is used, students should clearly show how they obtained the value of a single unit.

Students should then be able to add up the corresponding amounts and give a final answer.

Stage 2 Pictorial

Students should draw similar diagrams to the Physical Stage below in their books, this time using a bar model.

£3
$3.75

Students should clearly show how they are scaling up the value.

£3	£3	£1
$3.75	$3.75	$1.25

$$£1 = \frac{\$3.75}{3} = \$1.25$$

Students should then show the total value and clearly state their final answer.

£3	£3	£1
$3.75	$3.75	$1.25
$8.25		

£7 = $8.25

Stage 3 Semi-Abstract

At this stage students should use a proportion table to determine the cost of pens in Shop A.

÷3 ×7

Shop A

£	£3	£1	£7
$	$3.75	$1.25	$8.25

÷3 ×7

Students should then clearly state their final answer.

£7 = $8.25

Stage 4 Abstract

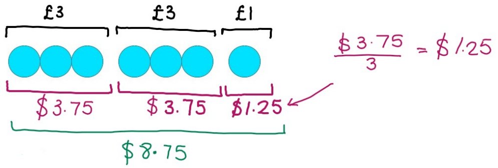
Students should represent the problem in a table.

Students should clearly show that the division operations need to be applied to both sides to represent the direct proportionality.

	£	:	$	
	£3	:	$3.75	
÷3	£1	:	$1.25	÷3
x7	£7	:	$8.25	x7

Students should then clearly state their final answer.

£7 = $8.25



£7 = $8.25

Curriculum Area:

GEOMETRY

Angles on Straight Lines and Points
Interior and Exterior Angles
Area of Polygons and Circles
Surface Area
Volume
Transformations

ANGLES – STRAIGHT LINES AND POINTS

GENERAL TEACHING & LEARNING POINTS	• Students should constantly be made to justify their answers. • Consistent terminology is essential. The terminology to be used is below.
VOCABULARY	**Corresponding angles are equal.** **Alternate angles are equal.** **Co-interior angles add up to 180°** **Vertically opposite angles are equal** **Angles on a straight line add up to 180°** **Angles around a point add up to 360°**

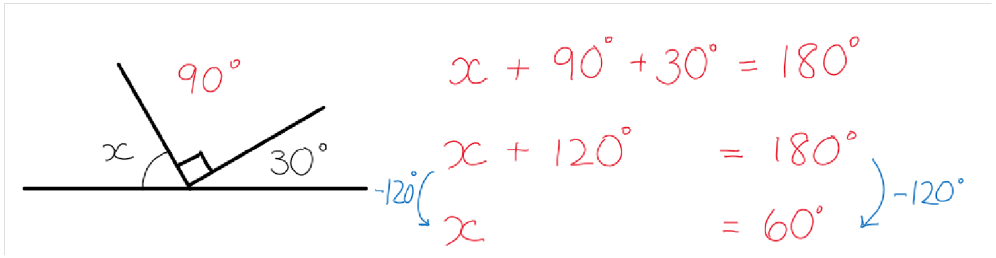
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students could discover the rules for angle problems by measuring angles when diagrams are drawn to scale.	Since this is a pictorial topic, these stages are combined. Students should form and solve an equation given angle facts.		

Example(s)

Example(s) - These should be seen in books

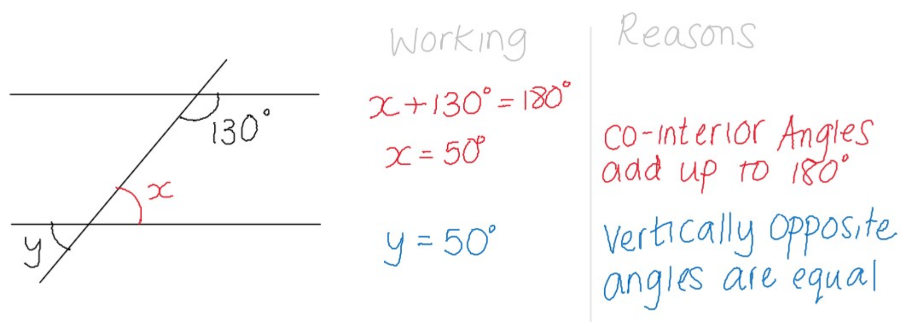
Students should form and solve an algebraic equation in order to determine the missing angle.

Work out the size of angle x.



Students could discover the rules for angle problems by measuring angles when diagrams are drawn to scale.

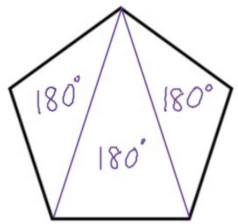
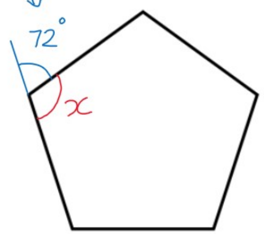
Work out the size of angle y. Give reasons at each stage of your working.



Students could be encouraged to split their page in half with one column as the working and the second for the reasons.

INTERIOR AND EXTERIOR ANGLES

GENERAL TEACHING & LEARNING POINTS	• Students should not purely be taught a formula, students should understand that the volume links to the number of layers of a shape • Students should then understand that the volume of a prism (and cylinder) is the Area of the Cross-Section x Length		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students could discover the rules for angle problems by measuring angles when diagrams are drawn to scale.	Students should determine the interior and exterior angles of a polygon by splitting the shape into triangles.	Students should understand the link between interior and exterior angles.	Students should understand and apply the formula $(n-2)\times 180°$
Example(s)	**Example(s) - These should be seen in books**		
	Determine the sum of the interior angles of a regular pentagon.		

Students could discover the rules for angle problems by measuring angles when diagrams are drawn to scale.	Students should pick a particular point and separate the polygon into triangles.  Pentagon = 3 triangles $= 3 \times 180°$ $= 540°$	Students should understand that interior and exterior angles sum to 180° and that the sum of the exterior angles of any polygon sum to 360°. $\frac{360°}{5} = 72°$  $x + 72° = 180°$ $x = 108°$ (−72° on both sides) Sum of interior Angles = $108° \times 5 = 540°$	Pentagon = 5 sides Sum of interior Angles $= (n-2) \times 180°$ $= (5-2) \times 180°$ $= 3 \times 180°$ $= 540°$

AREA OF POLYGONS AND CIRCLES

GENERAL TEACHING & LEARNING POINTS

- Students should be taught the areas of polygons in the following order:

  Rectangles → Parallelograms → Triangles → Trapeziums → Circles

- Students should NOT be merely taught a formula. Students should **understand** that each formula can be determined from the area of a parallelogram or rectangle

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**

Stage 1 Physical

Students could use physical versions of a rectangle or parallelogram to determine the formulae for the area of each shape.

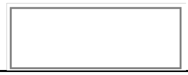
Stages 2–4

Students should be able calculate the area of a shape, including compound shapes.
Students should clearly state any formula and any values given in the questions.
For answers involving π, students should always state the exact answer prior to using any rounding.

Example(s)

Rectangle

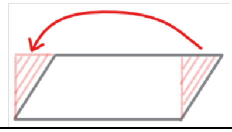
A = b x h OR A = bh



Parallelogram

A = b x h OR A = bh

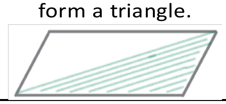
Students should move part of the parallelogram to the opposite side to form a rectangle.



Parallelogram

$A = \frac{b \times h}{2}$ OR $A = \frac{bh}{2}$

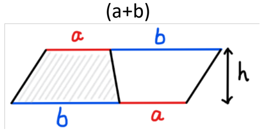
Students should split a parallelogram in half to form a triangle.



Trapezium

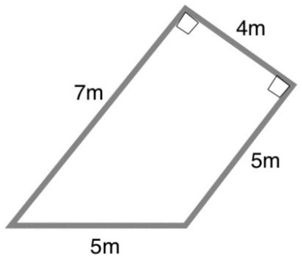
$A = \frac{a+b}{2} \times h$

Students should rotate two trapezia to identify the area is half a parallelogram with side length (a+b)



Example(s) - These should be seen in books

Calculate the area of this trapezium.



Students should clearly state the formula to be used.

$A = \frac{a + b}{2} \times h$

Students should state the required information from the diagram.

a = 5m b = 7m h = 4m

Students should then substitute into the formula

$A = \frac{5 + 7}{2} \times 4$

A = 6 x 4

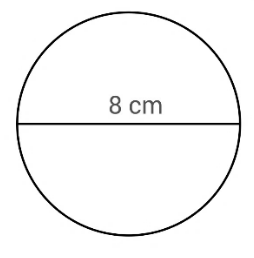
Students should ensure that the units are included in the final answer.

$A = 24m^2$

Calculate the area of this circle.

Students should clearly state the formula to be used.

$A = \pi r^2$



Students should state the required information from the diagram. d = 8cm so r = 4cm

Students should then substitute into the formula

$$A = \pi \times 4^2$$

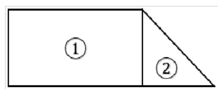
Students must include the answer in terms of π answer

$$A = 16\pi$$

Students should ensure that the units are included in the final answer.

$$A = 50.27cm^2$$

When working with compound shapes, students should clearly label each part of the shape. Students should then calculate each area separately as shown above.

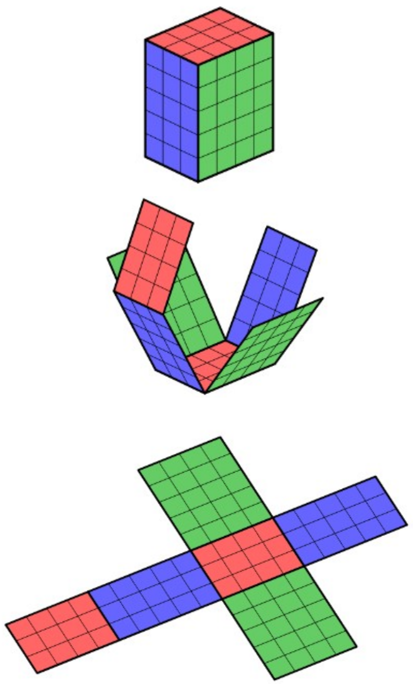
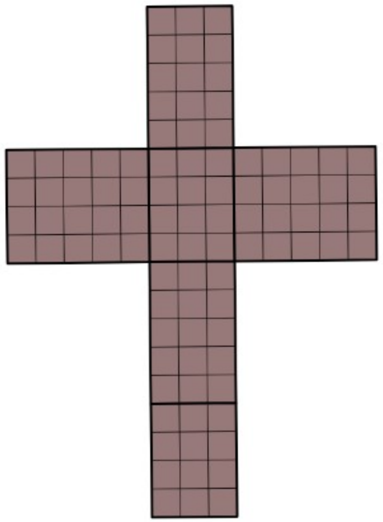


SURFACE AREA

GENERAL TEACHING & LEARNING POINTS	
	• Students should be encouraged to understand what is my meant by the surface area by investigating the nets of shapes. • Students should be able to find the surface area of cubes, cuboids, prisms and cylinders.

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should use the 3D Geometric Shapes to understand the different faces.	Students should use a net to find the surface area.	Students should list each of the faces then total.	Students should identify repeated faces.
Example(s)	**Example(s) - These should be seen in books**		

A cuboid has a length of 3cm, a width of 4cm and a height of 5cm. Work out the surface area of the shape.

Students should use the 3D Geometric Shapes to understand the different faces. The Maths Pad interactive tool could be used to help understanding. 	Students should draw a net of the cuboid using the squares in their books.  Surface Area = 94cm^2	Students should list all of the sides and calculate the areas of each. Students should then total these. Front 5 x 4 = 20 Back 5 x 4 = 20 Top 4 x 3 = 12 Bottom 4 x 3 = 12 Left 3 x 5 = 15 Right 3 x 5 = 15 Surface Area = 94cm^2	Students should identify the faces that have equal area. Students should then total these. Front 5 x 4 = 20 Top 4 x 3 = 12 Left 3 x 5 = 15 Total = 47 Surface Area = 2 x 47 = 94cm^2

Students will then be able to count the squares to find the Surface Area.			

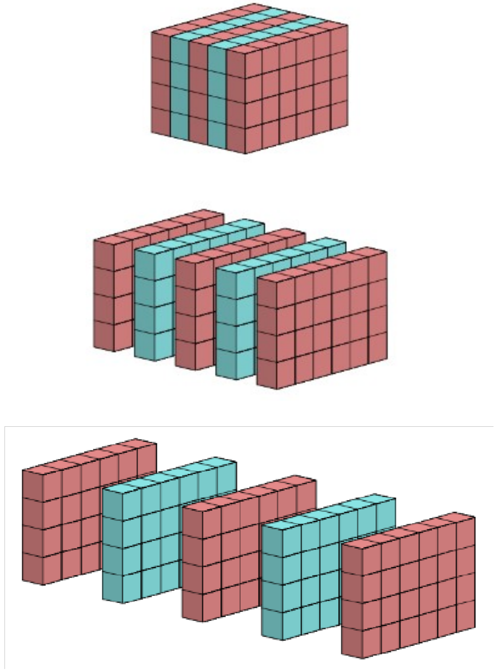
VOLUME

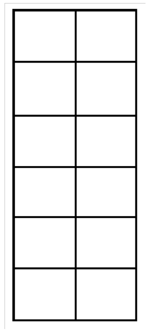
GENERAL TEACHING & LEARNING POINTS	• Students should not purely be taught a formula, students should understand that the volume links to the number of layers of a shape • Students should then understand that the volume of a prism (and cylinder) is the Area of the Cross-Section x Length		
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should use the 3D Geometric Shapes to understand the different faces.	Students should draw the cross section, find the area and multiply by the length.	Students should find the area of the cross section and multiply by the length.	Students should state and use a formula.
Example(s)	**Example(s) - These should be seen in books**		
	A cuboid has a length of 4cm, a idth of 2cm and a height of 6cm. Work out the volume of the shape.		

Students should use the 3D Geometric Shapes to understand the different faces and the idea of a cross-section.

The Maths Pad interactive tool could be used to help understanding.

Students should understand that the volume is the area of the cross-section multiplied by the number of layers.





Cross Section

$A = bh$

$A = 2 \times 6$

$A = 12cm^2$

Volume =
Area of Cross-Section X Length

$V = 12 \times 4 = 48cm^3$

Volume =
Area of Cross-Section X Length

Area of CS = $2 \times 6 = 12cm^2$

$V = 12 \times 4 = 48cm^3$

$V = lwh$

$V = 4 \times 2 \times 6 = 48cm^3$

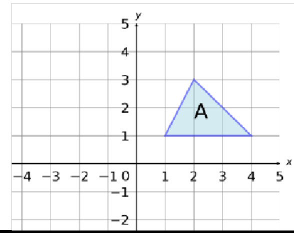
TRANSFORMATIONS

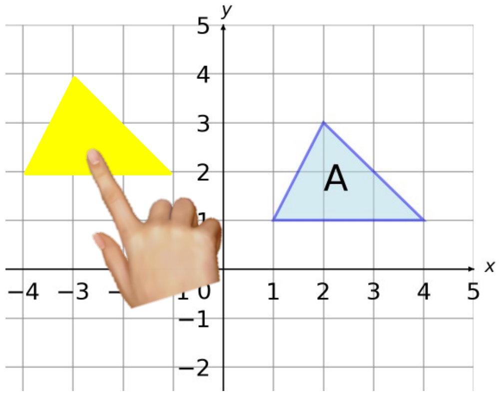
GENERAL TEACHING & LEARNING POINTS	• Students should understand the idea of a vector and writing and describing vectors prior to performing transformations • Students should understand the meaning of the terms Object and Image, and this terminology should be used throughout the unit.

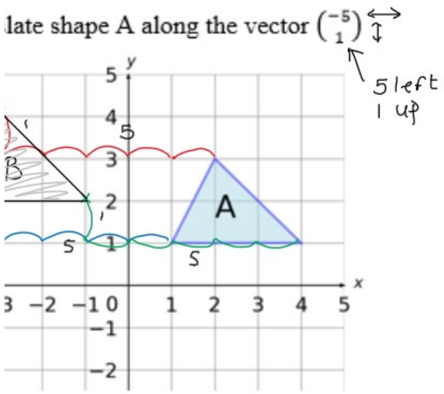
Translations

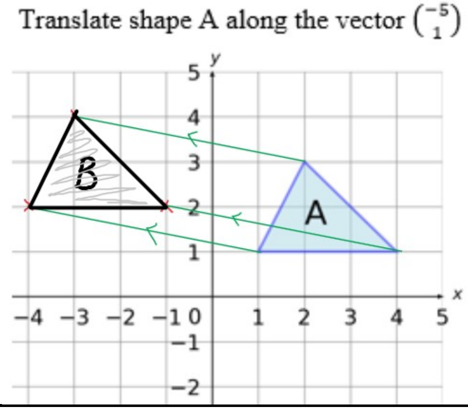
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should use a physical, cut out, shape to move the object to the image's location.	Students should draw on their grid how the shape will move. Students should do this from each point.	Students should draw the vector that is to be applied to each point.	Students should understand that a vector has a fixed origin and therefore the points can be determined through vector arithmetic.
Example(s)	**Example(s) - These should be seen in books**		

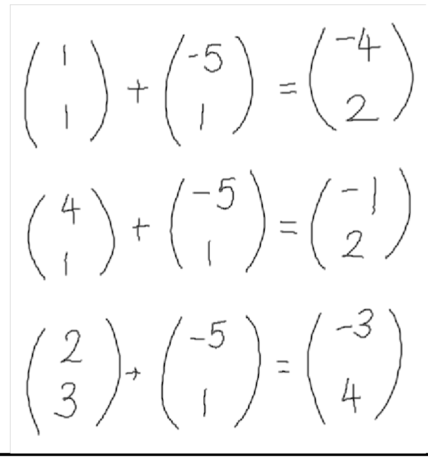
Translate shape A (shown on the right) along the vector 4(#$5







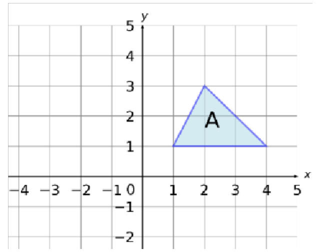




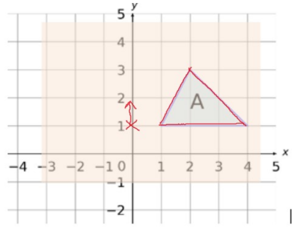
Reflections

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**

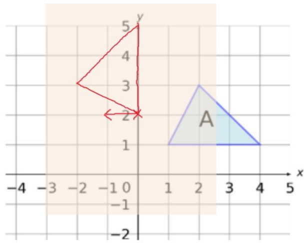
Due to the pictorial nature of reflections, the process outlined below satisfies the Physical, Pictorial and Semi-Abstract stages.

Example(s)	**Example(s) - These should be seen in books**
	Rotate shape A 90° anti-clockwise about the point (0, 1) 

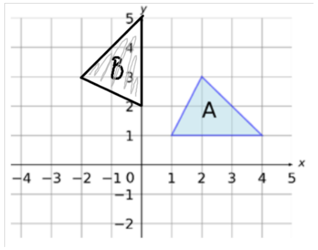
Students should clearly mark the centre of rotation and draw an upward arrow from it. Students should trace over the shape, as shown below.



Students should then rotate the tracing paper so that the arrow points in the required direction after the rotation and then trace over the shape.

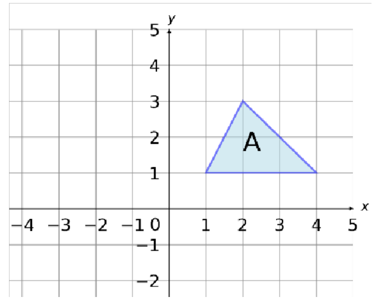


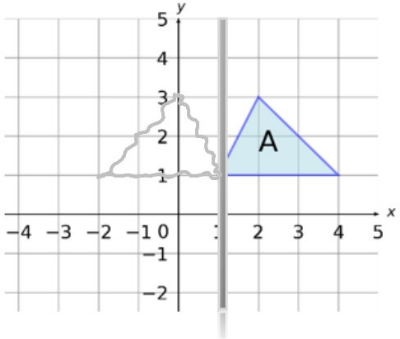
Students should ensure that their final image is drawn clearly with a ruler.



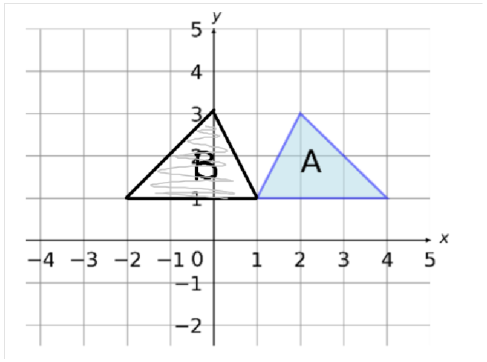
The full Abstract Stage of rotations requires knowledge of **Matrices** which is not covered at GCSE.

Reflections

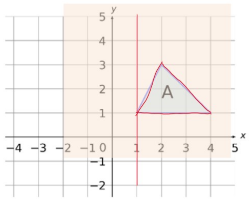
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should use a physical mirror to reflect the shape.	Students should use tracing paper to reflect the shape.	Students should draw the vector that is to be applied to each point.	Students should understand that a vector has a fixed origin and therefore the points can be determined through vector arithmetic.
Example(s)	**Example(s) - These should be seen in books**		
	Reflect the shape A in the line x = 1.		



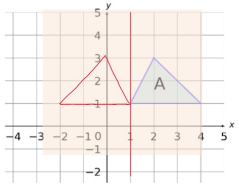
Students should then accurately draw the image with a ruler.



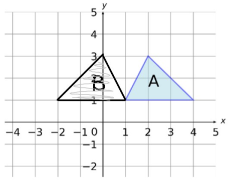
Students should use tracing paper to trace the shape **and** the mirror line.



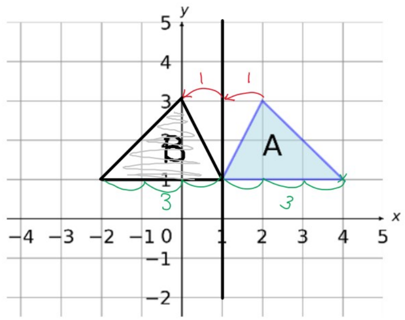
Students should flip the tracing paper over and line up the mirror line.



Students should then accurately draw the image with a ruler.

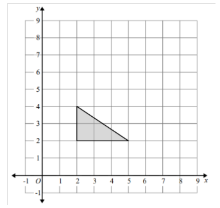


Students should be able to perform the reflection by determining how to get from each vertex of the shape and then performing the appropriate same step to get to the point on the image.

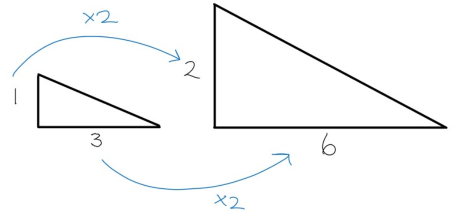


The full Abstract Stage of Reflections requires knowledge of **Matrices** which is not covered at GCSE.

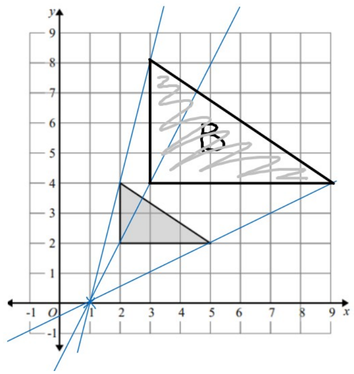
Enlargements

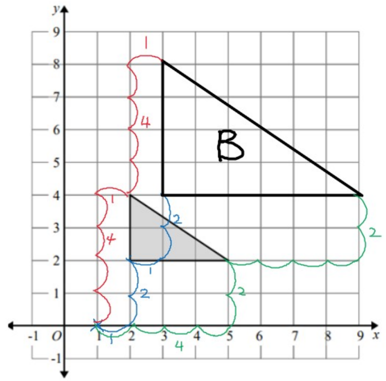
Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to determine the dimensions of a shape after an enlargement.	Students should use a ray diagram to draw the enlargement	Students should determine the vector from the centre of rotation to each vertex, then perform that vector the required number of times.	Students should be able to multiply the vector by the SF of the enlargement.
Example(s)	**Example(s) - These should be seen in books**		
	Enlarge the shape on the right by scale factor of 2, centred at (1, 0).		

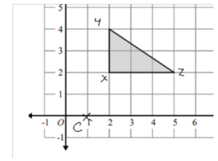
Students should be able to determine and draw the size of the enlarged shape.	Students should draw straight lines through each vertex of the shape and the centre of enlargement.	Students should determine the translation from the centre of enlargement to each vertex of the shape and repeat this the same number of times as the scale factor.	Students should label the centre of enlargement C and the vertices of the shape using different letters.



Students should understand what dimensions the enlarged shape will have.





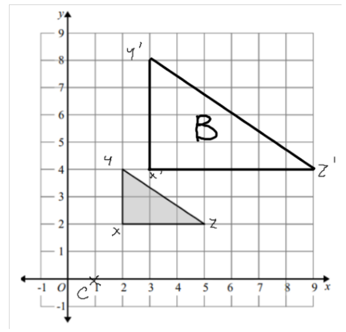


Students should then use scalar vector multiplication to determine the vector from the centre to that point on the image.

$\overrightarrow{CX} = \begin{pmatrix}1\\2\end{pmatrix}$ so $\overrightarrow{CX'} = 2\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}2\\4\end{pmatrix}$

$\overrightarrow{CY} = \begin{pmatrix}1\\4\end{pmatrix}$ so $\overrightarrow{CY'} = 2\begin{pmatrix}1\\4\end{pmatrix} = \begin{pmatrix}2\\8\end{pmatrix}$

$\overrightarrow{CZ} = \begin{pmatrix}4\\2\end{pmatrix}$ so $\overrightarrow{CZ'} = 2\begin{pmatrix}4\\2\end{pmatrix} = \begin{pmatrix}8\\4\end{pmatrix}$



Curriculum Area:

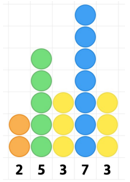
STATISTICS

Mean, Mode, Median and Range

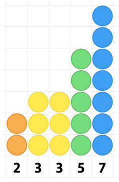
MEAN, MODE, MEDIAN AND RANGE

Stage 1 **Physical**	Stage 2 **Pictorial**	Stage 3 **Semi-Abstract**	Stage 4 **Abstract**
Students should be able to represent a problem using manipulatives (counters or multi-link cubes). It may be useful for students to use different colours for the different values.	Students should draw the manipulatives as a diagram in their books.	Students should be comfortable with the formal definitions of mode, mean, median and range at this point.	Students should be able to find the mean, mode, median and range without needing a diagram. Higher attaining students should link to algebra during working.
Example(s)	**Example(s) - These should be seen in books**		

Find the mode, range, median and mean of 2, 5, 3, 7, 3.



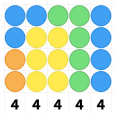
Students should line up the manipulatives in order of size from the beginning. I.e.



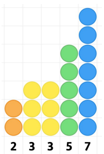
Mode
If different colours have been used this is the colour used for the most columns. If colours aren't used then students should use which height is the most common.

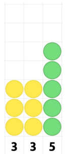
Range
Students should count the difference between the biggest value and the smallest value.

Mean
Students should aim to level out the manipulatives into a rectangle, removing one from the largest to the smallest. The mean is the height of each bar at the end of the process.
It is important to highlight that the mean can be a decimal at this point.

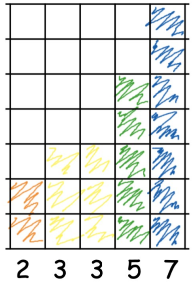


Median
Students should remove the smallest and biggest column and repeat. The median is the column that is remaining. Where there are two columns remaining students should find the mean of the two columns as above.





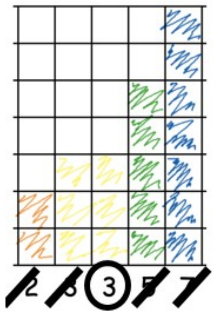
Students should represent the numbers as bars using the squares in their books. This should be smallest to largest.



Mode, Range and Mean

As Physical Stage.

Median
Students should cross off the numbers below the bars smallest then largest. Students should circle the median value.



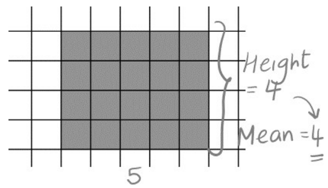
Mode
Students at this point should know the mode is the most common value and should be able to determine this by inspection from the list of numbers.

Range
Students at this point should know the range is
Largest - Smallest
Students should clearly show this calculation, i.e.

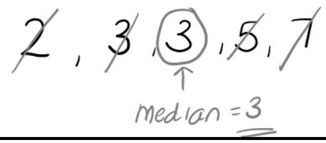
$$7 - 2 = 5$$

Mean
At this point students should have identified the relationship that the total number of blocks/counters forms the area of the rectangle and the mean is the height of the rectangle.

This should lead to the understanding that the mean is the total number divided by the amount of numbers.



Median
Students at this point should know the median in the middle number.
Students should be able to find the median without the aid of a diagram.
Students should cross off the numbers, smallest then largest.
Students should circle the median value.



Mode

As semi-abstract stage.

Range

As semi-abstract stage.

Mean
Students should be confident with the definition and calculating the mean as below:

$$\frac{2+3+3+5+7}{5}$$

Higher attaining students should be introduced to the formula written using sigma and notation, i.e.

Median
Students at this point should know the median in the middle number.
Students should be able to state which number is the middle number using the formula

$$\text{Median Number} = \frac{n+1}{2}$$

In this case:

$$\frac{n+1}{2} = \frac{5+1}{2} = 3$$

So the median is the 3rd number in the list when the values are put in order.